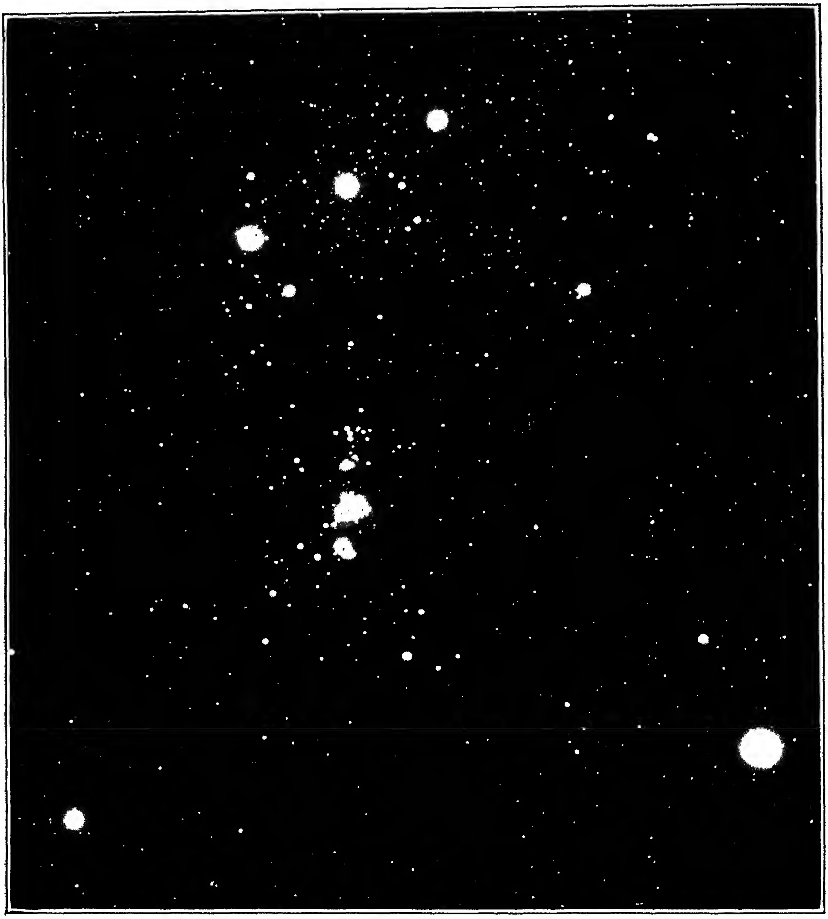

THE FIELD OF ORION

This photograph was taken at the Royal Observatory, Greenwich, with a four-inch portrait lens, the exposure being one hour fourteen minutes, and is reproduced by the courtesy of the Astronomer Royal. The Great Nebula in Orion lies directly below Anilam, the centre star of the 'belt'.

[*See* Chapter XXI

ADMIRALTY NAVIGATION MANUAL

VOLUME II

1938

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The Lords Commissioners of the Admiralty have decided that a new Admiralty Navigation Manual is required for the information and guidance of the Officers of His Majesty's Fleet ; for this purpose the present Manual has been compiled in three volumes under the direction of the Captain, H.M. Navigation School, Portsmouth, by the Staff of H.M. Navigation School assisted by the Dean and Staff of the Royal Naval College, Greenwich.

By the publication of this Manual, the Admiralty Manual of Navigation, 1928, is superseded and may be destroyed.

BY COMMAND OF THEIR LORDSHIPS.

*Admiralty, S.W.*1.
October, 1937.

PREFACE

The Admiralty Navigation Manual, 1938, consists of three Volumes :

Volume I is a practical guide for executive officers covering the syllabus laid down for examination in Navigation and Pilotage for the rank of Lieutenant, but omitting the study of nautical astronomy.

Volume II is the text book of nautical astronomy completing the above syllabus.

Volume III is based on the syllabus for officers qualifying in Navigation and deals solely with advanced subjects and mathematical proofs not included in Volumes I and II. It will be unnecessary for executive officers in general to study this Volume.

Thanks are due for the information given by the Hydrographic Department, the Naval and Marine Divisions of the Meteorological Office, various instrument manufacturers, and to Messrs. Rich and Cowan for the use of extracts from *On the Bridge*, by Captain J. A. G. Troup, R.N.

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CHAPTER I

POSITION ON THE EARTH'S SURFACE

The Earth's Shape. The Earth is an ellipsoid, the solid formed by rotating an ellipse about an axis, but for the purposes of navigation it may be considered a perfect sphere because the errors which result from this assumption are usually negligible. A slight flattening of the polar regions is the chief departure from the spherical shape. The effect of this flattening is explained fully in Volume III of this Manual. In this volume, which is concerned with practical navigation, it is dealt with only to the limited extent necessary for deriving the fundamental unit of measurement, and all other calculations are based on the supposition that the Earth is a perfect sphere.

The Earth's Movement. Since the position of any point on a sphere can be fixed most conveniently by measuring its distances from two axes on the sphere, the first problem in position-finding concerns the choice of these axes. The Earth's daily rotation governs this choice because it gives rise to the cardinal directions, east, west, north and south.

The Earth's movement has three components :

(1) a daily rotation about a diameter.

(2) a yearly passage along an elliptical orbit round the Sun.

(3) a translation through space shared by all members of the solar system.

The ordinary observer on the Earth can detect only the first two.

The Poles. In figure 1a, PP' is the diameter about which the Earth rotates. P and P', the points where this diameter meets the surface, are called the *poles*.

East and West. *East* is defined as the direction in which any point on the Earth's surface is moving as the result of the Earth's rotation. This direction—anti-clockwise to an observer looking down on the pole P—is shown by the arrows in figures 1a and 1b.

West is the direction opposite to, or 180° from, east.

North and South. The two poles are distinguished arbitrarily. The *north* pole (P) is said to be that pole which lies to the left of an observer who is facing east. *North* is therefore that direction in which an observer would have to move in order to reach the north pole, and it is clearly at right angles to the east–west direction. The other pole (P') is known as the *south* pole.

The Great Circle. A sphere is formed by rotating a semicircle about its diameter, and any section of a sphere by a plane is a circle. If the plane goes through the centre of the sphere, the resulting section is the largest that can be obtained and is known as a *great circle*. It is important because it gives the navigator the shortest

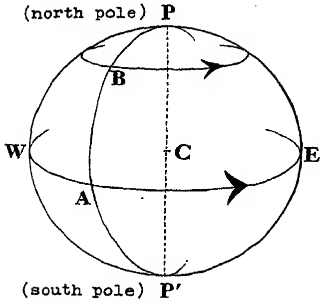


FIGURE 1a.

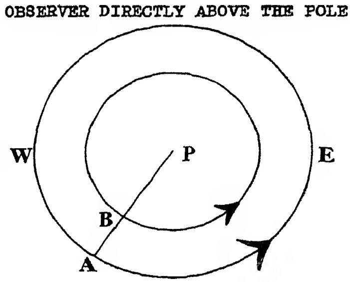


FIGURE 1b.

track between any two places that lie on it, and it is also the path of the ray which he detects when using a wireless installation for direction-finding.

The Small Circle. If the plane does not pass through the centre of the sphere, the section is known as a *small circle*. (Figure 2.)

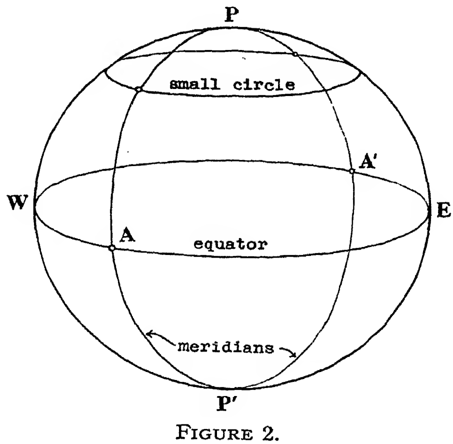


FIGURE 2.

The Equator. When the plane of section not only passes through the Earth's centre but is also perpendicular to the axis of rotation, it cuts the surface in a particular great circle known as the *equator*. Every point on the equator is therefore 90° from the poles.

The Meridians. Great circles joining the poles form what are known as meridians. In figure 2, PAP' is one meridian, and $PA'P'$, on the other side of the Earth, is a second. A meridian is thus a semi-great circle joining the poles.

UNITS OF MEASUREMENT

Angular Distance between Two Places. Since it is agreed to consider the Earth as a sphere, the distance between places on the Earth's surface is conveniently expressed in angular measure.

The length of the great-circle arc FT in figure 3, for example, is ($R \times$angle FCT), R being the radius of the sphere and the angle FCT being measured in radians. Because R is constant, the length FT is proportional to the size of the angle FCT. For this reason it is customary to refer to the shorter arc of the great circle joining two

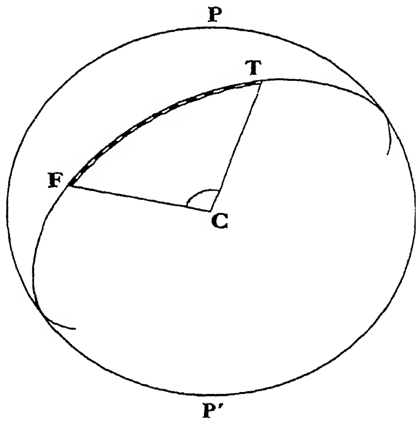


FIGURE 3.

points as the *angular distance* between them. The most convenient unit for measuring distance on the Earth's surface is therefore based on an angular unit, and the unit of distance selected is the length of a great-circle arc which subtends an angle of one minute at the centre of curvature.

Variation in the Length of One Minute of Arc. If the Earth were actually a perfect sphere, this length of great-circle arc would be constant everywhere, and there would be no difficulty in deciding its value.

In figure 4 the shape of the Earth is intentionally exaggerated to show the effect of the flattening at the poles, and it is seen that AB, the length of arc at the pole subtending an angle of one minute at O, is greater than $A'B'$, the length of arc along a meridian at the equator which also subtends an angle of one minute at O', O and O'

being the centres of curvature. The positions of O and O' are determined by the amount of bending in the arcs AB and $A'B'$. Only when the ellipse becomes a circle, and the curvature is the same at all points, will O and O' coincide with C, the centre of the ellipse.

The Geographical Mile. The ellipsoid, which is the Earth, is formed by rotating the semi-ellipse PQP' about its minor axis PP'. The major axis thus traces a circle of radius CQ, and every point on the equator is equidistant from the Earth's centre. The length of a minute of arc measured along the equator is therefore constant. and it is known as a *geographical mile.*

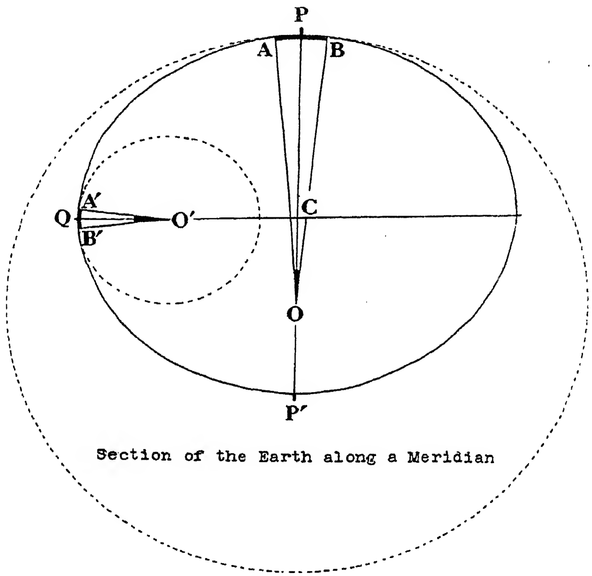


FIGURE 4.

Expressed in ' land ' units, the geographical mile is 6,087·2 feet, and the equator is 24,902·18 statute miles, a statute or ' land ' mile being an arbitrary unit introduced by Queen Elizabeth, who decreed that it should be ' 8 furlongs of 40 perches of 16½ feet ', 5,280 feet in all.

The statute mile is never used in navigation.

The Standard Nautical Mile. The nautical or ' sea ' mile is, strictly, the length of a minute of arc measured along a meridian, but this length varies from about 6,046 feet at the equator to about 6,108 feet at the pole, and it is thus unsuitable for a practical unit which must be constant. Its approximate mean value is therefore taken.

The unit employed in measuring distances at sea is the standard nautical mile *of* 6,080 *feet, and the term* mile *in navigation refers to that unit.*

Except on charts, where the symbol M is now adopted, it is always denoted by ′, which is also the symbol for a minute of arc. Thus *10′·8* means *10·8 standard nautical miles*, or, since the custom of speech omits the 'standard', simply *10·8 nautical miles.*

The symbol is placed before the decimal point and not after the final figure (10′·8 and not 10·8′) to ensure that no error is made in the position of the decimal point.

If, for example, the angle FCT in figure 3 is 56°25′·5, the length of the great-circle arc FT is also 56°25′·5 in angular measurement, or $(56\times60+25\cdot5)$ minutes of arc ; that is, 3,385·5 nautical miles.

The nautical mile being 6,080 feet and the statute mile 5,280 feet, 7 nautical miles are approximately equal to 8 statute miles. More accurately, 13 nautical miles are equivalent to 15 statute miles.

NOTE. The arithmetical labour of converting nautical miles into statute miles may be avoided by using the Traverse Table which is explained in Chapter III. If this table is entered for an angle of 30° and the figure against that angle in any latitude column is treated as the number of nautical miles to be converted, the distance given at the top of the column will be the corresponding number of statute miles.

The Cable. For measuring distances less than a mile, the unit employed is the *cable*, which is one-tenth of a nautical mile. Practice takes it to be 200 yards.

The Knot. In navigation, the unit of speed is one nautical mile per hour, and that unit is called a *knot*. If a vessel steams through the water a distance of 8×6,080 feet in one hour, she is said to be steaming at 8 knots, or simply, steaming 8 knots.

The term *knot* derives from the old method of finding a ship's speed through the water by means of a log and line. Equally spaced along the line were pieces of coloured rag tied in knots, and the number of these that paid out during a period set by a particular sand-glass when the log was thrown overboard, gave the ship's speed in nautical miles per hour. Simple arithmetic shows that if the sand-glass empties in 14 seconds, as the 'short log glass' did, the distance between two knots is just under 24 feet.

LATITUDE AND LONGITUDE

To find the position of any point in a plane, it is sufficient to know its shortest distances from two lines in that plane, preferably at right angles to each other. (Figure 5a.)

When the point lies on a sphere, the same method holds in principle, but the distances from the two axes must be measured in angular and not linear units.

The corresponding axes on the Earth's surface are the equator and the meridian through Greenwich. (Figure 5b.) The equator is

selected because it is conveniently situated for the purpose ; it is midway between the poles and its plane is at right angles to the axis of spin. The Greenwich meridian is arbitrarily selected, and is known as the *prime meridian.* Distances from these axes are

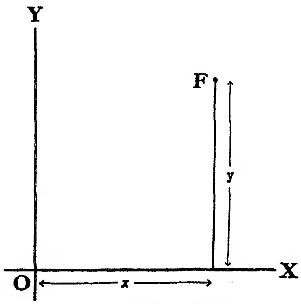


FIGURE 5a.

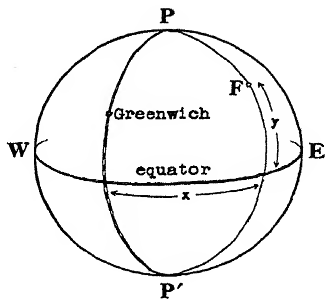


FIGURE 5b.

measured in the directions governed by the Earth's rotation—north, south, east and west.

Latitude. In figure 6, PFP' is the meridian passing through a place F and meeting the equator at L. The angular distance FL is

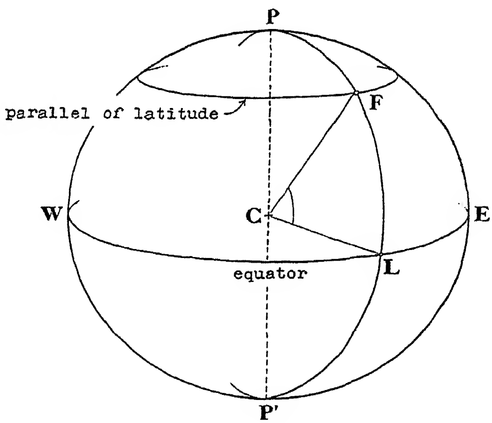


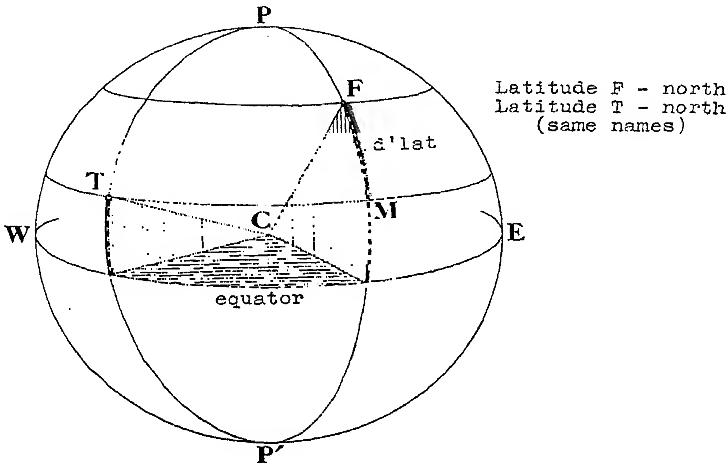
FIGURE 6.

called the latitude of F. The angle FCL thus measures the latitude of F.

The latitude of a place is therefore expressed in degrees and minutes, and it is said to be that number of degrees and minutes 'N.' or 'S.' according as the place lies north or south of the equator.

When it is expressed in minutes, it gives the distance FL in nautical miles. Thus, if the latitude of F is 50°30′N., then F is 3,030 nautical miles north of the equator.

Parallel of Latitude. Places having the same latitude as F



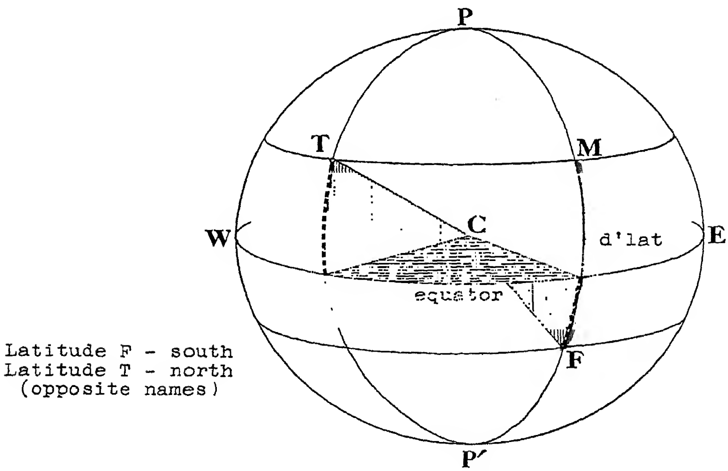


FIGURE 7a. FIGURE 7b.

clearly lie on a small circle, the plane of which is parallel to the plane of the equator. This small circle is called a *parallel of latitude*.

Difference of Latitude. The difference of latitude between two places, F and T, is the difference between the latitudes of F and T;

that is, the length *FM* along the meridian through *F*, cut off by the parallels of latitude through *F* and *T*.

If the two places are on the same side of the equator and have the same name—both are *north* in figure 7a—that difference is found by subtracting the latitude of *T*, the smaller one, from the latitude of *F*.

If *T* lies on the side of the equator opposite to *F*, and the two places have opposite names, *north* and *south*, as in figure 7b, the difference of latitude is the sum of the separate latitudes.

The expression *difference of latitude* is used whether the latitudes have the same names or not. It is always referred to as the *dee-lat*, an abbreviation which is conveniently written *d'lat*.

The rule for finding the d'lat may be summarised thus: *same names, subtract*; *opposite names, add.*

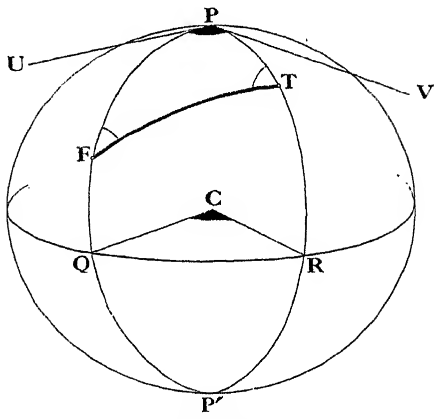


FIGURE 8.

If a ship is steaming from *F* to *T*, it is necessary to put a name to the d'lat in order to indicate whether she is moving north or south. In figure 7a she would be moving south, and the d'lat would be marked S. In figure 7b she would be moving north, and the d'lat would be marked N.

For example, a ship steaming from Portsmouth (50°48′N.) to Gibraltar (36°07′N.) would change her latitude by 14°41′S. because she herself is moving south. Thus:

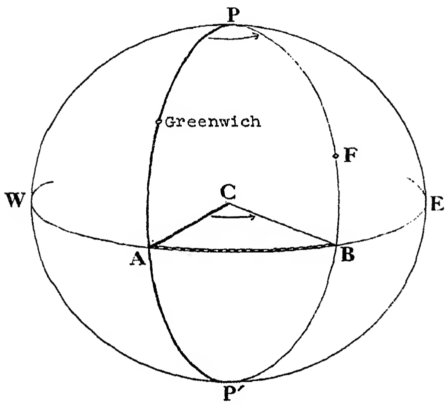
From Portsmouth	50°48′N.
To Gibraltar	36°07′N.
d'lat (same names, subtract)	14°41′S.

A ship steaming from Suva, in Fiji, (18°09′S.) to Honolulu

(21°18′N.) would change her latitude by 39°27′N. because she herself is steaming north. Thus :

From Suva	18°09′S.
To Honolulu	21°18′N.
d'lat (opposite names, add)	39°27′N.

The Spherical Triangle. Figure 8 shows that the great-circle arc FT forms with the meridians through F and T a triangle PFT,



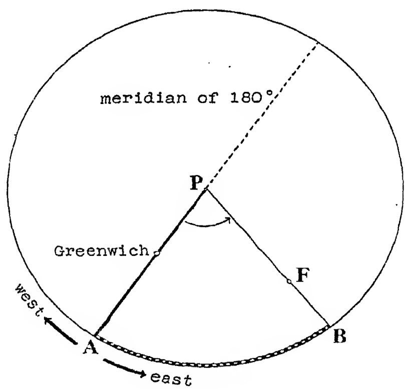
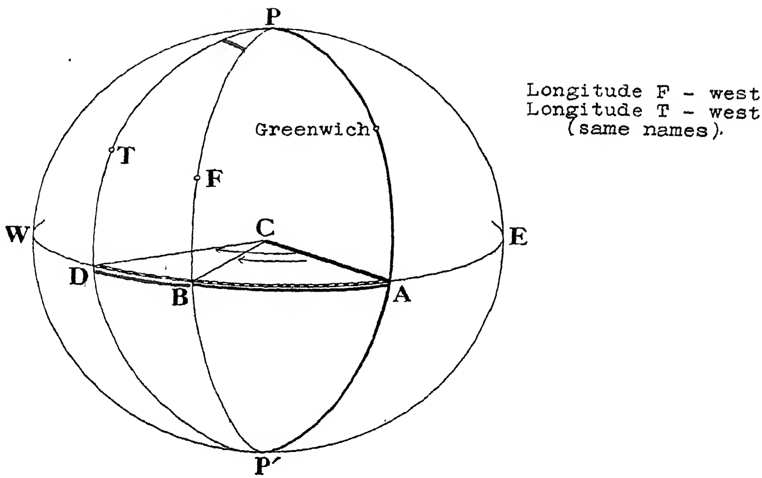


FIGURE 9a. FIGURE 9b.

each side of which is part of a great circle. This triangle is known as a *spherical triangle* because it lies wholly in the surface of a sphere. There is no restriction placed upon the positions of F and T, but, since the sides of the triangle are formed by the shorter arcs of the

great circles passing through its vertices, no side or angle can be greater than 180°.

Spherical Angles. These, in a spherical triangle, correspond to



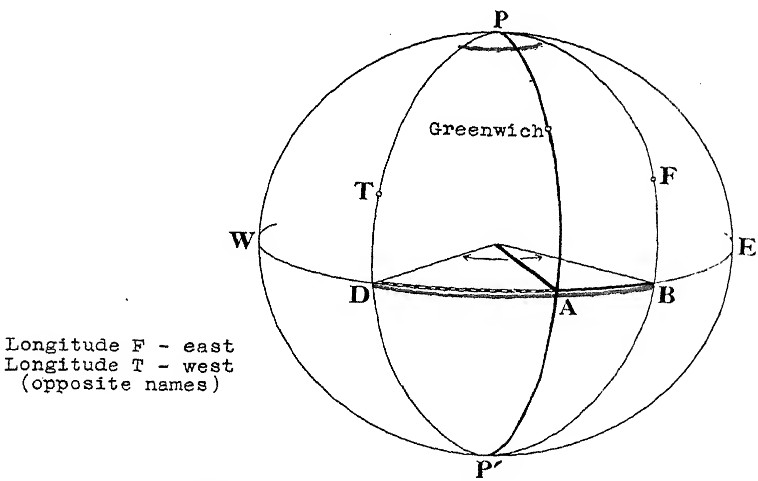


FIGURE 10a. FIGURE 10b.

the angles at the vertices of a plane triangle, and they are the angles between the great circles forming the sides.

The planes of the great circles $P'FP$ and $P'TP$ cut the tangent plane at P in UP and VP. These lines are therefore tangents to

the two great circles, and the angle between them measures the angle between the great circles.

When the two great circles are meridians, as in figure 8, the angle between them is the angle at the pole, and, since the tangent plane at the pole is parallel to the plane of the equator, this angle, UPV, is equal to the angle QCR. The angle at the pole is therefore conveniently measured by the equatorial arc QR lying between the meridians.

Longitude. In figure 9a, PFP' is the meridian through F meeting the equator in B. The meridian through Greenwich meets the equator in A. The angular distance AB is called the longitude of F. The longitude of a place is thus the angle between the meridian through the place and the meridian through Greenwich.

Longitude is named 'E.' or 'W.' according as the place is east or west of the Greenwich meridian.

Longitude cannot be greater than 180° east or west because the plane of the Greenwich meridian bisects the Earth. For this reason longitude is always measured along the shorter arc—AB, that is, and not $AWEB$.

Difference of Longitude. The difference of longitude between two places, F and T in figures 10a and 10b, is the difference between the longitudes of F and T; that is, the length BD cut off along the equator by the meridians through F and T, or the angle at the pole between the meridians of the two places.

The expression *difference of longitude* is always referred to as the *dee-long* and may be conveniently written *d'long*, and it is apparent that the same rule holds for finding the d'long as for finding the d'lat: *same names, subtract; opposite names, add.*

A ship, for example, steaming from Sandy Hook, New York, (74°00'W.) to the Bishop Rock Light in the Scillies (6°27'W.) would change her longitude by 67°33'E. because she herself is moving east. Thus:

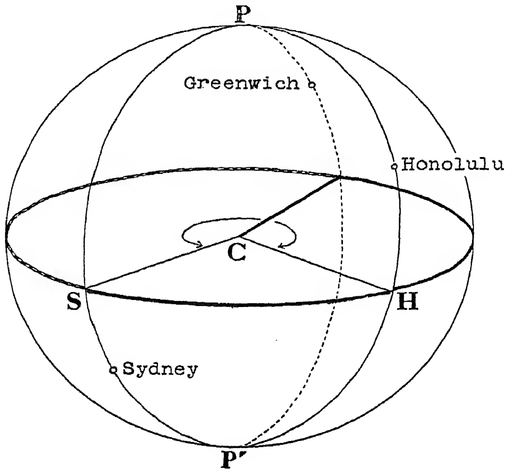
From Sandy Hook	74°00'W.
To Bishop Rock	6°27'W.
d'long (same names, subtract)	67°33'E.

A ship steaming from Malta (14°31'E.) to Gibraltar (5°21'W.) changes her longitude by 19°52'W. because she herself is moving west. Thus:

From Malta	14°31'E.
To Gibraltar	5°21'W.
d'long (opposite names, add)	19°52'W.

Should this sum exceed 180°, a small adjustment is necessary, as the following example will make clear.

Suppose that a navigator wishes to take his ship from Sydney (151°13′E.) to Honolulu (157°52′W.). By rule, the d'long is



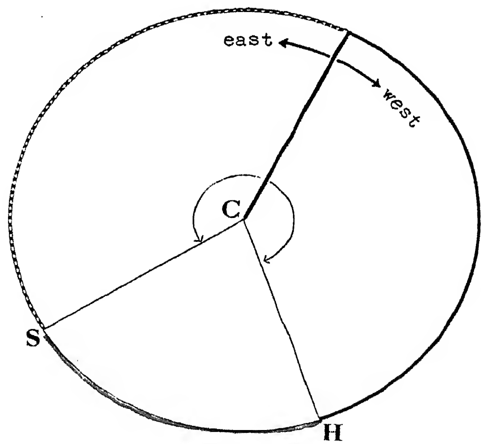


FIGURE 11a. FIGURE 11b.

309°05′W., because he would go back to the Greenwich meridian and then on, proceeding west all the time. But in practice he would obviously go the shorter way, which is east. The number obtained

from the rule must therefore be subtracted from 360° if it exceeds 180°, and its name must be reversed. Thus :

From Sydney	151°13′E.
To Honolulu	157°52′W.
d'long (opposite names, add)	309°05′W.
	360°
d'long	50°55′E.

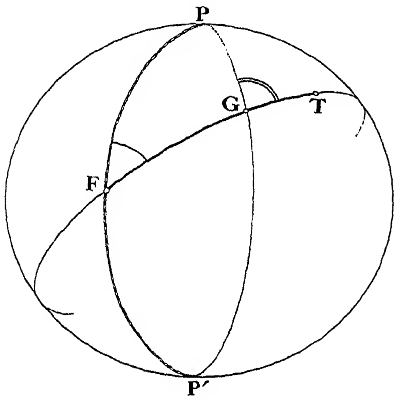
The position of any place on the Earth is thus defined by its latitude and longitude, and the position of any other place can be defined in relation to the first by a d'lat and a d'long.

CHAPTER II

COURSE AND BEARING

A distinction should always be drawn between *direction* and *bearing* in order to avoid looseness of expression.

Direction. Before a navigator can take his ship from Scapa Flow to Bergen, say, he must first ascertain the direction in which Bergen



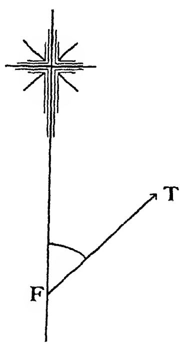


FIGURE 12a.

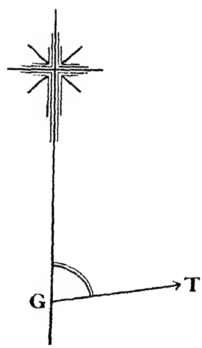


FIGURE 12b.

lies from Scapa Flow, so that by steering in this direction he may arrive at Bergen. Direction is thus determined by the point on the horizon towards which a person is aiming or a ship is moving. It is thus essentially a line.

As shown in the last chapter, the cardinal directions are north, south, east and west.

Bearing. To a navigator in mid-ocean, the horizon is a circle drawn about the ship as centre, and there is nothing to distinguish one point on that circle from another. In order that he may steer in a particular direction he must therefore be able to refer to some datum line or fixed direction. The *angle* between this datum line and the direction in which he wishes to steer is called the *bearing* of the point for which he is making.

True Bearing. The most convenient datum is the meridian through the navigator's position, because that is the north–south line. Bearings measured from this datum are known as *true bearings.*

In figure 12a, FP, the meridian through F, gives the direction of true north. FT, the great circle joining F to T, gives the direction of T. The angle PFT is the true bearing of T from F.

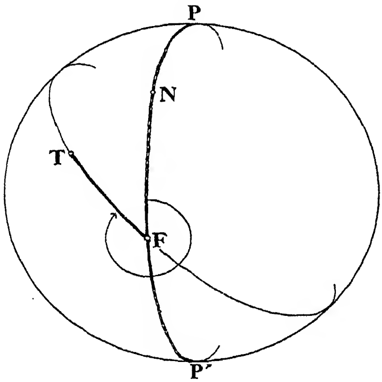


FIGURE 13a.

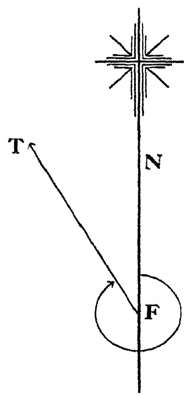


FIGURE 13b.

At any intermediate point G between F and T the true bearing of T is the angle PGT, and this is not equal to the angle PFT. To an observer moving along the great circle from F to T, the true bearing of T changes continuously. Only when T is close to F may this change be neglected. The area of the Earth's surface traversed by FT is then sufficiently small to be considered as a plane or flat surface on which great circles appear as straight lines.

The angle PFT is always measured *clockwise* from 0° to 360°, and it is always written as a three-figure number in order that the bearing may be recognised immediately as a true one. In figures 12a and 12b, T would be said to bear 043° from F, and 080° from G.

In figures 13a and 13b, T bears 330° from F, and the point N, which lies on the meridian through F and therefore due north of F, bears 000°.

Position of Close Objects. It is often convenient to indicate the position of an object by its distance and bearing from a known or key position. A shoal, for example, would be said to be 7 miles, 229°, from a certain lighthouse, and a cruiser 10 miles, 078°, from a flagship.

True Course. The direction in which a ship moves in still water is the direction of her fore-and-aft line. The angle between this fore-and-aft line and the meridian through her position is called her *true course.*

If the ship is following the great-circle track between two places, the true bearing of her head is both her course and the true bearing of her destination, and, since the latter changes continuously between the two places, she must alter course continuously if she is to keep to the great circle.

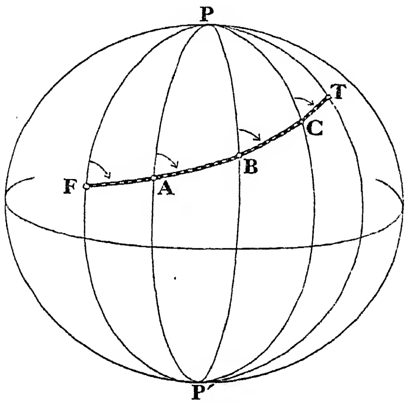
The Compass. The navigational compass is an instrument that gives the necessary datum line from which courses and bearings can be measured.

Compasses are of two kinds—gyro and magnetic. The *gyro compass* derives its directive force from the gyroscope which is its essential component, and it points to or near to the *true north.* The *magnetic compass* secures its directive force from the Earth's magnetism, and it points to or near to the *magnetic north.* Both compasses, with their errors and corrections, are described fully in Volume I.

CHAPTER III

THE RHUMB LINE

The most convenient course for a ship to steer is a steady course, one, that is, along which the bearing of her head remains constant. Her track must then cut all the meridians at the same angle, and in general it will spiral towards the nearer pole.



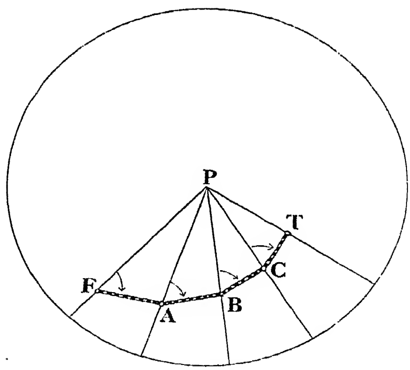


FIGURE 14a. FIGURE 14b.

The Rhumb Line. A line on the Earth's surface which cuts all the meridians at the same angle is called *a rhumb line*.

In figures 14a and 14b, $FABCT$ is the rhumb line joining F to T.

The angles PFA, PAB, PBC and PCT are all equal, and any one of them may be taken as the course.

When this constant angle is 90°, the rhumb line is a parallel of latitude or the equator. (Figure 15.) When the angle is 0°, the rhumb line coincides with the meridian. The track of a ship steering a steady course is thus either a rhumb line or a meridian, and except at the poles (where the meridians cut at angles ranging from 0° to 180°) a meridian may be considered to be a rhumb line.

Because of the predominance of the rhumb line in navigation, it is customary to talk about the *rhumb-line distance* between F and T as *the distance*, and mention of the distance between two places will, unless qualified, always refer to the distance along the rhumb line joining them, measured in nautical miles.

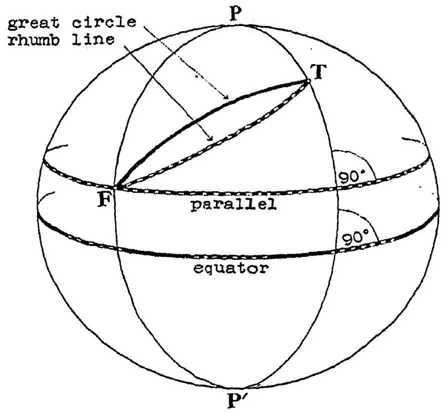


FIGURE 15.

In the same way the *rhumb-line course* is always referred to as *the course*, and mention of the course that must be followed in order to go from one place to another will always refer to the angle between any meridian and the rhumb line joining them.

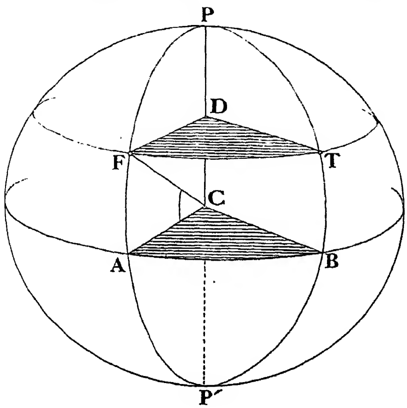
Distance along a Parallel of Latitude. In order to find the course and distance along a rhumb line, it is necessary to know the distance along a parallel of latitude between two given points because the formulæ for rhumb-line sailing are built up by considering a large number of small right-angled triangles, in each of which one side lies along a parallel of latitude.

In figure 16a, FT is the arc of the parallel of latitude, the length of which is to be found. FT may thus be taken as the distance along the parallel between the meridians through F and T.

AB is the distance along the equator between the same meridians; that is, the d'long between F and T.

It is also apparent that the nearer the parallel is to the pole—the higher the latitude, in other words—the shorter FT becomes.

But the d'long does not alter. FT must therefore bear to AB some relation depending on the latitude.



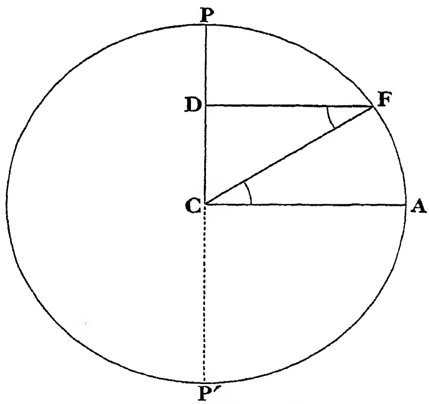


FIGURE 16a. FIGURE 16b.

To find this relation, consider the sections DFT and CAB. They are parallel and equiangular. Hence:

$$\frac{FT}{AB}=\frac{DF}{CA}$$

But, from the triangle DCF in figure 16b :

$$DF=CF \cos \text{(latitude)}$$
$$=CA \cos \text{(latitude)}$$

—since CF is equal to CA, each being a radius of the Earth. Therefore :

$$\frac{FT}{AB}=\frac{CA \cos \text{(latitude)}}{CA}$$

i.e. $$FT=AB \cos \text{(latitude)}$$

The distance along a parallel of latitude in nautical miles is thus equal to the d'long, expressed in minutes of arc, multiplied by the cosine of the latitude.

Suppose, for example, the latitude of the parallel is 40°N., and longitudes of F and T are 15°E. and 60°E. respectively. Then the d'long is 45° or, in minutes of arc which are also nautical miles at the equator, 2,700. Therefore :

$$\begin{aligned} FT &= 2{,}700' \times \cos 40° \\ &= 2{,}068'\cdot 3 \end{aligned}$$

Had the latitude been 60° instead of 40°, the distance along this new parallel would have been 2,700′ ×cos 60°, which is 1,350′.

Departure. The distance along a parallel of latitude is a particular example of what is called *departure.*

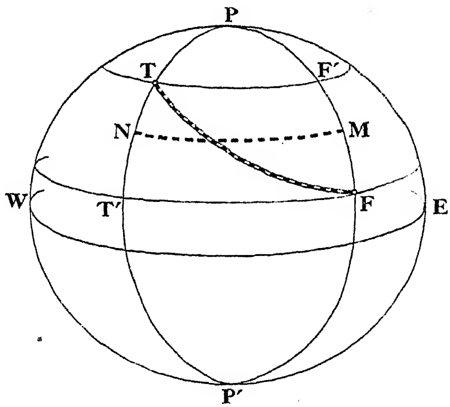


FIGURE 17.

Departure is the distance made good in an east–west direction in sailing from one place to another along a rhumb line.

Suppose that a navigator goes from F to T in figure 17.

The distance he moves in an east–west direction in doing so cannot be greater than FT', the distance along the parallel through F, because the two meridians FF' and $T'T$ converge north of FT'.

For the same reason it cannot be less than $F'T$.

The distance he moves in an east–west direction in going from F to T must therefore be equal to the distance along some parallel, MN, lying between the parallels through F and T.

Middle Latitude. The latitude of this parallel, MN, is called the *middle latitude* between F and T. Hence, by the formula just established :

$$\text{departure} = \text{d'long} \cos (\text{middle latitude})$$

The mathematical theory by which this middle latitude is found is given in Volume III. An approximation, however, suggests itself.

Mean Latitude. Except when the difference of latitude is large or the latitudes themselves are high, the middle latitude may be taken as the arithmetic mean of the two latitudes without appreciable error. The accurate formula :

$$\text{departure} = \text{d'long} \cos (\text{middle latitude})$$

—then becomes the approximate one :

$$\text{departure} = \text{d'long} \cos (\text{mean latitude})$$

When the two places are close to the equator, but on opposite sides of it, the departure can be taken as being equal to the d'long. If they are not sufficiently close for this assumption to be made, the arithmetic mean of the latitudes is no longer an approximation to the middle latitude, and the methods of Mercator sailing (Chapter V) must be used.

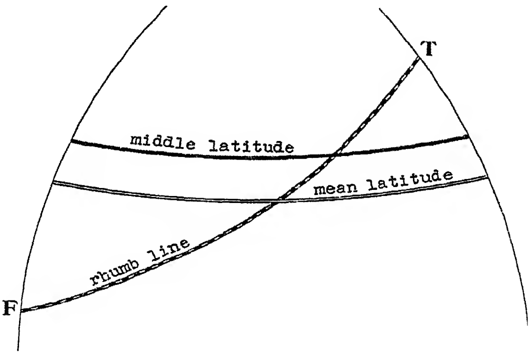


FIGURE 18.

In general, the use of a mean instead of a middle latitude suffices up to distances of 600′, the limit of the traverse tables.

To find the Middle Latitude. When the positions involved suggest that the middle latitude should be used, it can be found by adding a small correction (figure 18) to the mean latitude. This correction is given in *Inman's Tables*.

NOTE. *Inman's Tables* use the term 'middle latitude' for what is here referred to as the 'mean latitude', and 'true middle latitude' for what is here referred to as the 'middle latitude'.

If, for example, a ship steams from a position F in latitude 30°N., longitude 40°W., to a position T in latitude 34°N., longitude 36°W., her departure (easterly) by the approximate formula would be

$$\begin{aligned} & (40^\circ - 36^\circ) \cos \cdot \ (34^\circ + 30^\circ) \\ & = 240' \cos (32^\circ) \\ & = 203' \cdot 5 \end{aligned}$$

The correction from *Inman's Tables*, which is added to the mean latitude of 32° in order to give the middle latitude, is 3′. The accurate formula therefore gives the ship's departure as:

$$240' \cos(32°3')$$
$$=203'{\cdot}4$$

The difference between these two results is negligible, but if the ship had steamed from 50°N., 20°W., to 70°N., 8°W., say, it would have been appreciable. Her departure by the approximate formula would have been:

$$(20°-8°) \cos \tfrac{1}{2}(70°+50°)$$
$$=720' \cos(60°)$$
$$=360'$$

The accurate formula gives it as:

$$(20°-8°) \cos(60°+1°12'{\cdot}5)$$
$$=720' \cos(61°12'{\cdot}5)$$
$$=346'{\cdot}8$$

THE RHUMB-LINE FORMULÆ

With his knowledge of the distance along a parallel of latitude and the departure between two places, the navigator can find the course he must steer in order to follow the rhumb line joining two places, and also the distance he travels while doing so.

In figure 19, FT is the rhumb line, divided into a large number of equal parts FA, AB, BC . . .

Af, Ba, Cb . . . are the arcs of parallels drawn through A, B, C . . ., and the angles at f, a, b . . . are therefore right angles.

If the divisions of FT are made small enough, the triangles FAf, ABa, BCb . . . are themselves sufficiently small to be treated as plane triangles.

Also, since the course angle at F, A, B, C . . . remains constant by the definition of a rhumb line, these small triangles are equal.

Note. It should be clearly understood that to regard certain small triangles as plane is not to disregard the initial decision to consider the Earth as a sphere. The triangles are merely assumed to be sufficiently small to lie flat on the Earth's surface. This assumption, incidentally, gives rise to the expression *plane sailing*, which is popularly referred to as if *plane* were spelt *plain* and the sailing were free from difficulty.

Departure=Distance × Sin (Course). In covering the length AB, say, the navigator makes good in the west–east direction a distance which may be taken as being equal to aB since AB itself is small. That is, the smaller the triangles FAf, ABa, BCb . . . are taken, the more nearly does aB approximate to the departure between A and B.

The departure between F and T is therefore the sum of all these small arcs of parallels, fA, aB, bC . . .

But :

$$fA = FA \sin (\text{course})$$
$$aB = AB \sin (\text{course})$$
$$bC = BC \sin (\text{course})$$

Therefore, by addition :

$$fA + aB + bC + \ldots = (FA + AB + BC + \ldots) \sin (\text{course})$$

i.e. $$\text{departure} = \text{distance} \sin (\text{course})$$

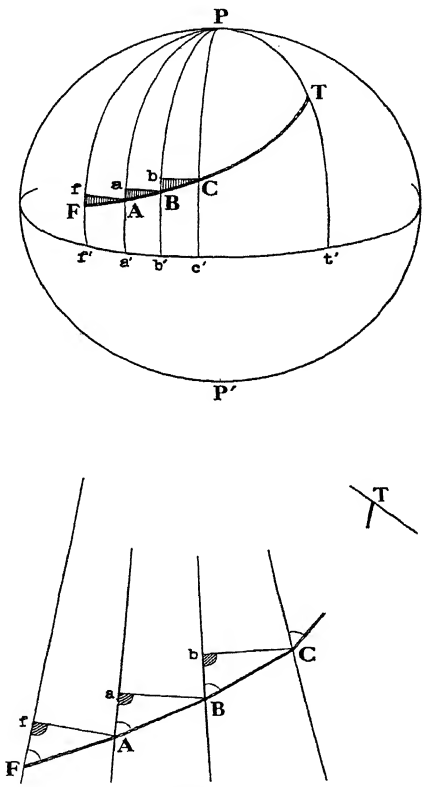


FIGURE 19.

D'lat = Distance × Cos (Course). This, the second rhumb-line formula, is found by considering the lengths of the sides of these small triangles that lie along the meridians. Thus :

$$Ff = FA \cos (\text{course})$$
$$Aa = AB \cos (\text{course})$$
$$Bb = BC \cos (\text{course})$$

But Ff is the d'lat between F and A ; Aa between A and B . . . so that $Ff+Aa+Bb+$ . . . is the total d'lat between F and T. Therefore, by addition :

$$Ff+Aa+Bb+\ldots=(FA+AB+BC+\ldots)\cos(\text{course})$$

i.e.

$$\text{d'lat}=\text{distance}\cos(\text{course})$$

Short-Distance Sailing. By the term short-distance sailing is meant the following of a rhumb-line track for a distance not greater than 600′. If this is done, the navigator can obtain all he wants to know about the track from the three formulæ :

$$\text{departure}=\text{d'long}\cos(\text{mean latitude}) \quad . \quad . \quad (1)$$
$$\text{departure}=\text{distance}\sin(\text{course}) \quad . \quad . \quad . \quad . \quad (2)$$
$$\text{d'lat}=\text{distance}\cos(\text{course}) \quad . \quad . \quad . \quad . \quad (3)$$

The course is given by (2) divided by (3). Thus :

$$\frac{\text{departure}}{\text{d'lat}}=\tan(\text{course})$$

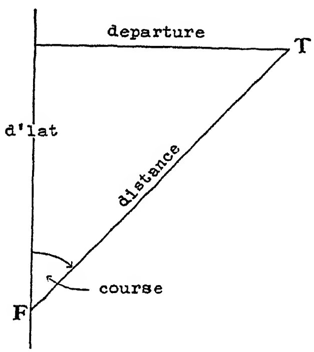


FIGURE 20.

In short-distance sailing, the navigator's problem usually takes one of two forms :

(1) *Required the course and distance when the positions of the starting point and destination are given.*

Since the d'long and the mean latitude are known, the departure can be found from formula (1).

Formula (4) now gives the course, and either (2) or (3) gives the distance.

(2) *Required the position after a given rhumb-line track has been followed for a given distance.*

In this problem, course and distance are known, and the departure and d'lat can be found at once from formulæ (2) and (3).

With this d'lat, the mean latitude is found, and formula (1) gives the d'long.

The Traverse Table. It should be apparent from their construction that these formulæ for short-distance sailing do no more than solve an ordinary right-angled triangle, the hypotenuse of which is the distance, the vertical side the d'lat, and the horizontal one the departure. (Figure 20.) To enable the navigator to solve this triangle quickly, he is provided with a *traverse table*.

Inman's Tables include a traverse table in which the d'lat and departure are given for any distance up to 600′, and for any angle up to 90°, and the extract printed here shows the arrangement of the table.

TRAVERSE TABLE

Distance	361		362		363		364		365		366		
CO.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	CO.
	COS.	SIN.	COS.	SIN.	COS.	SIN.	COS.	SIN.	COS.	SIN.	COS.	SIN.	
1°	360·9	6·3	361·9	6·3	362·9	6·3	363·9	6·4	364·9	6·4	365·9	6·4	89°
2	360·8	12·6	361·8	12·6	362·8	12·7	363·8	12·7	364·8	12·7	365·8	12·8	88
3	360·5	18·9	361·5	19·0	362·5	19·0	363·5	19·1	364·5	19·1	365·5	19·2	87
—	...	...	...	...	...	...	...	...	...	...	...	...	—
24	329·8	146·8	330·7	147·2	331·6	147·6	332·5	148·1	333·4	148·5	334·3	148·9	66
25	327·1	152·5	328·1	153·0	329·0	153·4	329·9	153·8	330·8	154·2	331·7	154·6	65
26	324·5	158·3	325·4	158·7	326·3	159·1	327·2	159·6	328·1	160·0	329·0	160·4	64
27	321·7	163·9	322·5	164·4	323·4	164·8	324·3	165·3	325·2	165·7	326·1	166·2	63
28	318·7	169·5	319·6	170·0	320·5	170·4	321·4	170·9	322·2	171·4	323·1	171·8	62
29	3 15·7	175·0	316·6	175·5	317·5	176·0	318·3	176·5	319·2	177·0	320·1	177·4	61
30	312·6	180·5	313·5	181·0	314·4	181·5	315·2	182·0	316·1	182·5	317·0	183·0	60
—	...	...	...	...	...	...	...	...	...	...	...	...	—
44	259·7	250·8	260·4	251·5	261·1	252·2	261·8	252·9	262·6	253·6	263·3	254·3	46
45	255·3	255·3	256·0	256·0	256·7	256·7	257·4	257·4	258·1	258·1	258·8	258·8	45
	SIN.	COS.	SIN.	COS.	SIN.	COS.	SIN.	COS.	SIN.	COS.	SIN.	COS.	
CO.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	DEP.	Diff. LAT.	CO.

The navigator who runs 363′ on a course 027°, for example, can see at once that his d'lat is 323′·4 and his departure 164′·8.

Had his course been 063°, the complementary angle, he would have read his departure and distance in the columns named at the bottom of the page, instead of at the top, for angles named on the right-hand side, instead of the left; and he would have found his d'lat to be 164′·8 and his departure 323′·4. That is because the triangle has been turned round, as shown in figures 21a and 21b.

The navigator who follows a course lying between 000° and 090° is moving both north and east. His d'lat is thus north, and his departure east. When he follows a course between 090° and 180°, his d'lat is south, though his departure remains east. These facts indicate that before the traverse table can be used, the course must be expressed in relation to the appropriate cardinal points.

If, for example, he followed a course of 243° for 363′, he would find his d'lat and departure by entering the table for a course of S.63°W. This tells him that his d'lat (164′·8) is south, and his departure (323′·4) is west.

Further Use of the Traverse Table. Since the traverse table does no more than multiply a number by the sine or cosine of an angle, thus :

$$363 \cos 27° = 323{\cdot}4$$
$$363 \sin 27° = 164{\cdot}8$$

—it can be used for changing departure into d'long or d'long into departure, the formula by which the change is made being :

$$\text{departure} = \text{d'long} \cos \text{(mean latitude)}$$

It is therefore sufficient to treat the d'long as ' distance ', and the mean latitude as ' course '.

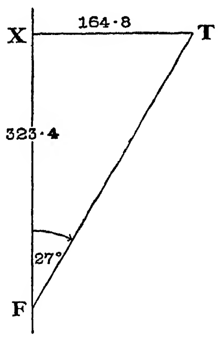


FIGURE 21a.

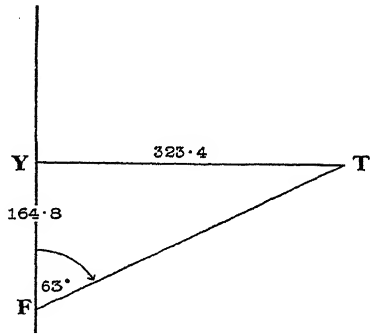


FIGURE 21b.

To change d'long into departure.

Against a course-angle equal to the mean latitude, read the departure in the cosine part of the column headed by a ' distance ' equal to the d'long.

To change departure into d'long.

This is the more usual problem, and it is solved by searching until, against a course-angle equal to the mean latitude, the number equal to the departure is found in the cosine part of the distance column. As before, the ' distance ' is the d'long.

When either conversion is made, it should be borne in mind that, since the d'long is equal to the departure multiplied by the secant of an angle, the number expressing the departure can never be greater than the number expressing the d'long. When that angle, given by the mean latitude, is zero, the departure is equal to the d'long, but for all other values of the mean latitude, the departure is less than the d'long.

It should also be borne in mind that, whereas the d'long is fundamentally an angle measured in minutes of arc, departure is a distance measured in nautical miles.

Inman's Tables include a special conversion table, arranged to convert departure into d'long. It comes after the traverse table, and it covers angles from 4° to 59° and distances from 1′ to 100′. It is a good plan, however, to use the traverse table as often as possible because a facility in using it can be acquired only by practice, and the table lends itself to the solution of any problem involving a right-angled triangle.

Method of Using the Rhumb-Line Formulæ. The following examples illustrate the method of using the formulæ for short-distance sailing.

(1) *A ship is required to steam from a position* F *in latitude* 35°52′*N., longitude* 3°06′*W., to a position* T *in latitude* 38°38′*N., longitude* 1°42′*E. What course must she steer, and what distance will she cover?*

latitude *F*	35°52′N.	longitude *F*	3°06′W.
latitude *T*	38°38′N.	longitude *T*	1°42′E.
d'lat	2°46′N.	d'long	4°48′E.
	=166′N.		=288′E.

The mean latitude is therefore 37°15′N. and the departure 229′E.

By traverse table

course =N.54°E.=054°
distance=283′

(2) *A ship leaves a position* F *in latitude* 41°05′*N., longitude* 2°12′*E., and steams a distance of* 305′ *on a course of* 115°. *What is her position at the end of the run?*

A course of 115° is S.65°E.
By traverse table :

d'lat =128′·9S.
departure=276′·4E.
d'long =360′·7E.

latitude *F*	41°05′·0N.	longitude *F*	2°12′·0E.
d'lat	2°08′·9S.	d'long	6°00′·7E.
latitude *T*	38°56′·1N.	longitude *T*	8°12′·7E.

The position at the end of the run is thus :

38°56′·1N.
8°12′·7E.

CHAPTER IV

THE GREAT CIRCLE

A straight line is the shortest distance between two points, and when the two points lie on a sphere, the arc of the great circle joining them is the curve that most nearly approaches the straight line because it has the greatest radius and therefore the least curvature. The shorter arc of the great circle joining two places on the Earth's surface is thus the shortest route between them, and the problem of finding this shortest distance is the problem of finding the length of a great-circle arc which is the side opposite the pole in the spherical triangle *PFT*. (Figure 22.)

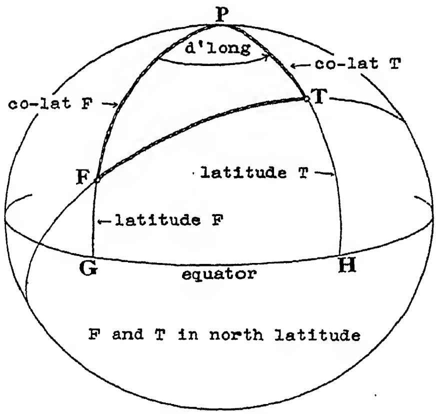


FIGURE 22.

The angle at the pole in this triangle is the d'long between *F* and *T*, and the angles at *F* and *T* are respectively the bearings of *T* from *F* and *F* from *T*.

The lengths of the sides *PF* and *PT* clearly depend upon the latitudes of *F* and *T*. When these latitudes have the same name —both are north in figure 22—*PF* is (90°—latitude *F*) and *PT* is (90°—latitude *T*). This distance, (90°—latitude), is known as the co-latitude of the place.

The Solution of the Spherical Triangle. There are six things to know about a spherical triangle: the sizes of its three angles, and the lengths of its three sides. Various formulæ connect these

angles and sides so that, if sufficient of them are given, the rest can be found. The common problems are those of finding the third side when two sides and their included angle are known, and a particular angle when the three sides are known.

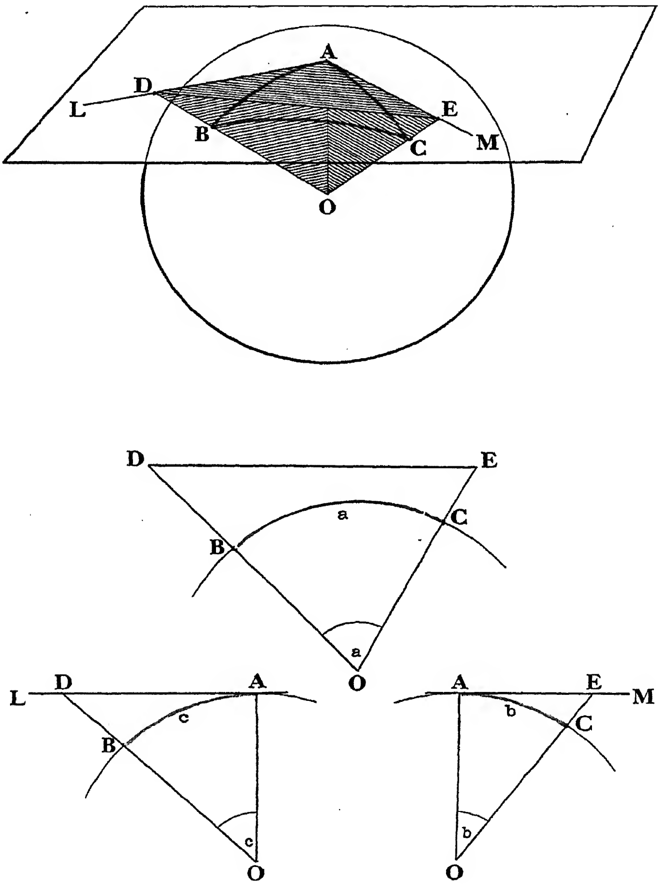


FIGURE 23.

The Fundamental or Cosine Formula. Figure 23 shows any spherical triangle ABC. It is customary to refer to its angles as A, B and C, and to the sides opposite these angles as a, b and c.

The angle between any two curves is the angle between their tangents at the point of section. The angle A is therefore the angle between the tangents to the great circles BA and CA at their point of section A.

Since both these tangents AL and AM are at right angles to the same radius OA, the plane in which they lie is the tangent plane at A, and the angle A in the spherical triangle is equal to the angle LAM in the tangent plane. But AL, being a tangent to AB, must also lie in the plane through the centre O, cutting the surface of the sphere in AB. Therefore, if OB is produced, it must cut AL in D. Similarly OC produced cuts AM in E.

There are thus four plane triangles, ADE, ODE, OAD and OAE, the last two of which are right-angled. Also, since a great-circle arc is measured by the angle it subtends at the centre, the angle DOE is a, the angle EOA is b, and the angle DOA is c.

The two right-angled triangles give :

$$\frac{AD}{OD}=\sin c \qquad \frac{AE}{OE}=\sin b \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{OA}{OD}=\cos c \qquad \frac{OA}{OE}=\cos b \quad . \quad . \quad . \quad . \quad (2)$$

$$OD^2=OA^2+AD^2 \qquad OE^2=OA^2+AE^2$$

i.e. $$OD^2-AD^2=OA^2 \qquad OE^2-AE^2=OA^2 \quad . \quad . \quad (3)$$

The other two triangles give :

$$DE^2=AD^2+AE^2-2\,AD.AE\cos A$$
$$DE^2=OD^2+OE^2-2\,OD.OE\cos a$$

(For the proof of these statements, see page 243 of the Appendix.)

Hence, by subtraction and substitution :

$$AD^2+AE^2-2\,AD.AE\cos A=OD^2+OE^2-2\,OD.OE\cos a$$

i.e. $$2\,OD.OE\cos a=(OD^2-AD^2)+(OE^2-AE^2)+2\,AD.AE\cos A$$

$$\cos a=\frac{OA}{OE}\times\frac{OA}{OD}+\frac{AE}{OE}\times\frac{AD}{OD}\cos A$$

This, from equations (1) and (2), may be written :

$$\cos a=\cos b\cos c+\sin b\sin c\cos A$$

Similarly, by working from the vertices B and C instead of A :

$$\cos b=\cos c\cos a+\sin c\sin a\cos B$$
$$\cos c=\cos a\cos b+\sin a\sin b\cos C$$

Thus, if any two sides and their included angle are given, the third side can be found, this side being the one opposite the only spherical angle in the formula.

The Haversine. As they stand, the above formulæ are not suitable for logarithmic work because the cosine of an angle between 90° and 180° is negative. To make them suitable, a function of the angle called a *haversine* is employed.

This function is half the versine—hence the name haversine—and since :

$$\text{versine } A=1-\cos A$$

—it follows that :

$$\text{haversine } A=\tfrac{1}{2}(1-\cos A)$$

The haversine of an angle is thus always positive, and it increases from 0 to 1 as the angle increases from 0° to 180°.

Inman's Tables include values of the haversine for values of A between 0° and 180°. For values of A greater than 180°, it should be remembered that :

$$\text{haversine } A = \text{haversine } (360° - A)$$

e.g. $$\text{hav } 210° = \text{hav } 150°$$

The proof of this adjustment is given on page 245 of the Appendix.

The Haversine Formula. To express the fundamental formula in terms of haversines instead of cosines, substitute for the appropriate cosines their values in terms of the haversine. Thus cos A can be written $(1-2 \text{ hav } A)$, and the formula becomes :

$$\cos a = \cos b \cos c + \sin b \sin c(1-2 \text{ hav } A) .$$

i.e. $$\cos a = \cos b \cos c + \sin b \sin c - 2 \sin b \sin c \text{ hav } A$$
$$= \cos (b \sim c) - 2 \sin b \sin c \text{ hav } A$$

The difference sign $\sim$, *used in cos* (b$\sim$c), *is always taken to mean that :*

(1) *the smaller quantity is subtracted from the greater when the quantities have the same sign ; both, for example, being derived from the latitudes of places north of the equator.*

(2) *the two quantities are added when they have opposite signs ; that is, for example, when one place is north of the equator and the other south.*

Similar substitutions for cos a and cos $(b \sim c)$ give :

$$1-2 \text{ hav } a = 1-2 \text{ hav } (b \sim c) - 2 \sin b \sin c \text{ hav } A$$

i.e. $$\text{hav } a = \text{hav } (b \sim c) + \sin b \sin c \text{ hav } A$$

This is the *haversine formula*—or, as it is sometimes called to distinguish it from its logarithmic counterpart, the *natural haversine formula*—and, as already shown, it is suitable for logarithmic work since all the quantities involved are positive and less than unity. The term ($\sin b \sin c$ hav A) is therefore less than unity, and this further simplifies the use of the formula because the anti-logarithm of the term can be found immediately from the haversine tables where natural and logarithmic values are printed side by side. (Line 5 in the working of the example that follows is simply the anti-logarithm of line 4.) The formula is thus conveniently arranged in two parts, one using logarithmic functions and the other natural haversines.

It is required to find b *when* a *is* 40°, c 75° *and* B 56°.

The appropriate formula is :

$$\text{hav } b = \text{hav } (c \sim a) + \sin c \sin a \text{ hav } B$$

—and in the evaluation of it *the known angle is written down first.* The actual work is arranged as shown.

$B=56°$	log hav	9·343 22
$c=75°$	log sin c	9·984 94
$a=40°$	log sin a	9·808 07
	log sin c sin a hav B	9·136 23
	sin c sin a hav B	0·136 84
$c\sim a=35°$	hav $(c\sim a)$	0·090 42
	hav b	0·227 26

The side b is therefore equal to $56°56'·5$.

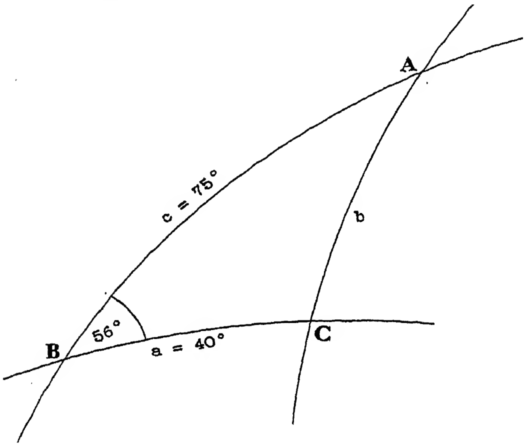


FIGURE 24.

The Half Log Haversine Formula. This formula, which gives one of the angles when the three sides are known, is derived from the fundamental formula by making substitutions similar to those used in building the haversine formula.

As before, the first substitution gives :

$$\cos a=\cos (b\sim c)-2 \sin b \sin c \operatorname{hav} A$$

i.e.

$$2 \sin b \sin c \operatorname{hav} A=\cos (b\sim c)-\cos a$$

By the rule for the subtraction of two cosines (see page 244 of the Appendix) this equality becomes :

$$2 \sin b \sin c \operatorname{hav} \mathrm{A}=2 \sin \tfrac{1}{2}[a+(b\sim c)] \sin \tfrac{1}{2}[a-(b\sim c)]$$

Therefore, by division :

$$\operatorname{hav} A=\operatorname{cosec} b \operatorname{cosec} c \sin \tfrac{1}{2}[a+(b\sim c)] \sin \tfrac{1}{2}[a-(b\sim c)]$$

But, from the definition of the haversine :

$$\operatorname{hav} x=\tfrac{1}{2}(1-\cos x)=\tfrac{1}{2}[1-(1-2 \sin^2 \tfrac{1}{2}x)]=\sin^2 \tfrac{1}{2}x$$

Therefore, by analogy :

$$\sin \tfrac{1}{2}[a+(b\sim c)]=\sqrt{\text{hav }[a+(b\sim c)]}$$
$$\sin \tfrac{1}{2}[a-(b\sim c)]=\sqrt{\text{hav }[a-(b\sim c)]}$$

By substitution :

$$\text{hav } A=\text{cosec } b \text{ cosec } c\sqrt{\text{hav }[a+(b\sim c)] \text{ hav }[a-(b\sim c)]}$$

In logarithmic form this is :

$$\begin{aligned}\log \text{hav } A &= \log \text{cosec } b\\ &\quad +\log \text{cosec } c\\ &\quad +\tfrac{1}{2}\log \text{hav }[a+(b\sim c)]\\ &\quad +\tfrac{1}{2}\log \text{hav }[a-(b\sim c)]\end{aligned}$$

Inman's Tables give the values of the ' half log haversine ', from which the formula takes its name, for values of the angle between 0° and 180°.

The following example shows the method of using the formula, and it should be noted that *the formula selected for giving the required angle must always start with the sides enclosing that angle.*

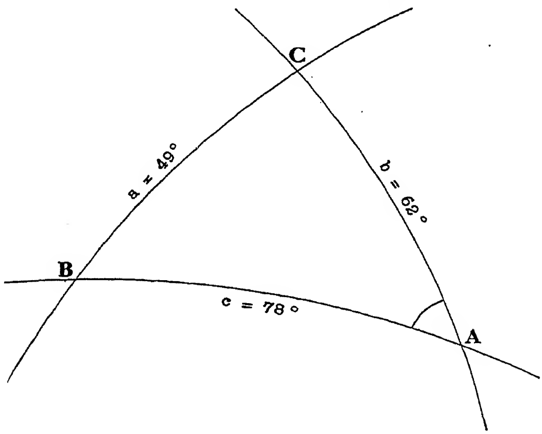


FIGURE 25.

It is required to find the angle A *when* a *is* 49°, b 62° *and* c 78°.

The appropriate formula is the one given above, and the work is arranged thus :

$c=78°$	log cosec c	0·009	60
$b=62°$	log cosec b	0·054	07
$b\sim c=16°$			
$a=49°$			
$a+(b\sim c)=65°$	½ log hav 65°	4·730	22
$a-(b\sim c)=33°$	½ log hav 33°	4·453	34
	log hav A	9·247	23

The angle A is therefore 49°42′·8.

If the angle B were required, the appropriate formula would be :

$$\begin{aligned}\log \text{hav}\, B = &\log \text{cosec}\, c\\ &+\log \text{cosec}\, a\\ &+\tfrac{1}{2}\log \text{hav}\,[b+(c\sim a)]\\ &+\tfrac{1}{2}\log \text{hav}\,[b-(c\sim a)]\end{aligned}$$

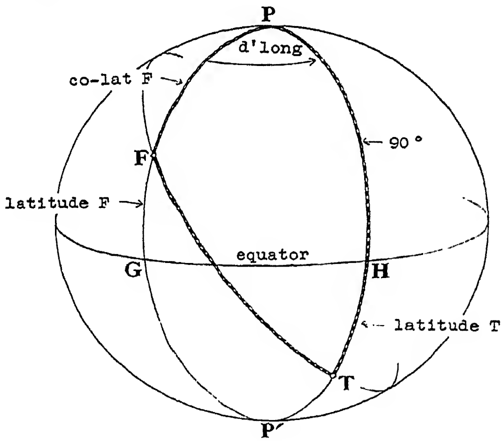
Great-Circle Distance and Bearing. If the particular triangle PFT is chosen instead of any triangle ABC, F and T being known places, then :

For latitudes of the same name. (Figure 22, F and T both north.)

$PF = 90° -$ latitude $F =$ co-latitude F
$PT = 90° -$ latitude $T =$ co-latitude T
angle $FPT =$ d'long

For latitudes of opposite name. (Figure 26, F north and T south.)

$PF = 90° -$ latitude $F =$ co-latitude F
$PT = 90° +$ latitude T
angle $FPT =$ d'long



F (north) and T (south) in opposite hemispheres
FIGURE 26.

If the starting point F is in the southern hemisphere, P' is substituted for P. The same relations hold.

Adapted to great-circle sailing, the haversine formula thus becomes :

$$\begin{aligned}\text{hav}\, FT = &\text{hav}\,[(90° - \text{lat. } F) \sim (90° \pm \text{lat. } T)]\\ &+ \sin(90° - \text{lat. } F)\sin(90° \pm \text{lat. } T)\,\text{hav}\,(\text{d'long})\end{aligned}$$

It will be noticed that the last term of this formula can be written :

$$\cos(\text{lat. } F)\cos(\text{lat. } T)\,\text{hav}\,(\text{d'long})$$

—and this is the form in which it is always used when applied to astronomical sights. Its use in this form, however, is not recom-

mended until the reader has made himself thoroughly familiar with the actual triangle.

The following examples illustrate the procedure :

(1) ***F* in north latitude, *T* in south.** F *is a position off Cadiz in latitude* 36°*N*., *longitude* 7°*W*., *and* T *is off Monte Video in latitude* 35°*S*., *longitude* 56°*W*. *What is the great-circle distance between them, and what is the bearing of* T *from* F *?*

The haversine formula gives :

d'long $= 49°$	log hav	9·235	45
$PF = 54°$	log sin	9·907	96
$PT = 125°$	log sin	9·913	36
		9·056	77
		0·113	97
$PF \sim PT = 71°$		0·337	22
	hav FT	0·451	19

The angular distance FT is therefore equal to $84°23'·9$ or $5{,}063'·9$.

(Although this is the shortest distance between the two places, it is not appreciably shorter, for reasons explained in Chapter V, than the rhumb-line track which is normally followed by ships.)

The ' half log haversine ' formula gives :

$PF = 54°00'·0$	log cosec	0·092	0
$FT = 84°23'·9$	log cosec	0·002	1
$FT - PF = 30°23'·9$			
$PT = 125°00'·0$			
$PT + (FT - PF) = 155°23'·9$	½ log hav	4·989	9
$PT - (FT - PF) = 94°36'·1$	½ log hav	4·866	2
	log hav PFT	9·950	2

The angle PFT is therefore $141°36'$, measured from north to west, and the true bearing is $218\frac{1}{2}°$.

NOTE. In finding the bearing, four-figure logarithms are used instead of five-figure logarithms because, in practice, bearings are not required to the same degree of accuracy as distances.

(2) ***F* and *T* in south latitude.** F *is a position off Tasmania in latitude* 43°*S*., *longitude* 140°*E*., *and* T *is off Cape Horn in latitude* 56°*S*., *longitude* 70°*W*. *What is the great-circle distance between them ? Find also the bearing of* T *from* F *and the bearing of* F *from* T.

The haversine formula gives :

d'long = 150°	log hav	9·969 89
FP' = 47°	log sin	9·864 13
TP' = 34°	log sin	9·747 56
$FP' \sim TP'$ = 13°		9·581 58
		0·381 58
		0·012 82
	hav FT	0·394 40

The angular distance FT is therefore 77°48′·0 or 4,668′.

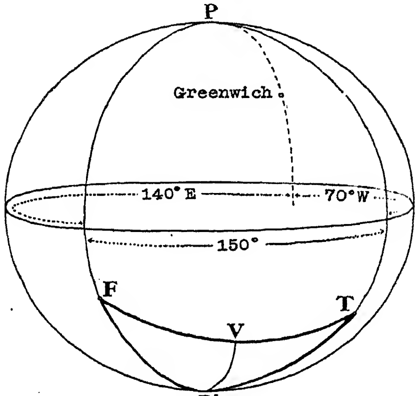


FIGURE 27.

(It can be shown by the methods of Chapter V that the rhumb-line distance between F and T is 5,850′, so that the great-circle distance is 1,182′, or some 20 per cent. shorter.)

The ' half log haversine ' formula gives :

(1) $P'F$	=47°00′	log cosec	0·135 9
FT	=77°48′	log cosec	0·009 9
$FT-P'F$	=30°48′		
$P'T$	=34°00′		
$P'T+(FT-P'F)$	=64°48′	½ log hav	4·729 0
$P'T-(FT-P'F)$	= 3°12′	½ log hav	3·445 9
		log hav $P'FT$	8·320 7

The angle $P'FT$ is therefore 16°38′, and the true bearing of T from F is 163½°.

(2)

$P'T$	$=34°00'$	log cosec	0·252 4
FT	$=77°48'$	log cosec	0·009 9
$FT-P'T$	$=43°48'$		
$P'F$	$=47°00'$		
$P'F+(FT-P'T)$	$=90°48'$	$\frac{1}{2}$ log hav	4·852 5
$P'F-(FT-P'T)$	$=3°12'$	$\frac{1}{2}$ log hav	3·445 9
		log hav $P'TF$	8·560 7

The angle $P'TF$ is therefore 21°59′, and the true bearing of F from T is 202°.

Great-Circle Sailing. From this example it is seen that a ship following the great-circle track between the given places changes her course from 163½° to 022°. That is, she gradually alters course through 141½°. But in practice any such continuous alteration of course is impossible. A ship must necessarily hold a steady course

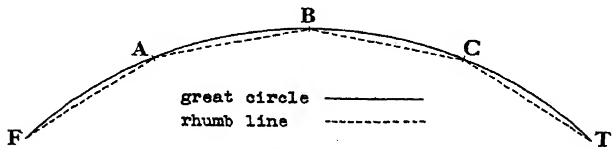


FIGURE 28.

(which means that she follows a rhumb line while doing so) until a definite alteration is made. The track which a navigator works out when he wishes to steam along a great circle is therefore a series of rhumb lines joining successive points on the great circle. This is known as approximate great-circle sailing or, simply, great-circle sailing.

Figure 28 (which is actually a mercatorial drawing—see Chapter V) shows this series for any great circle. The navigator would alter course at A, B and C, and he would choose the lengths FA, AB . . . to suit his convenience. FA, for example, might be a twelve-hour run.

Composite Track. Since the great-circle track between two places not on the equator passes nearer to the pole than the rhumb-line track, the ship may be carried into the ice region. To avoid this, and yet keep the distance to be steamed as short as possible, it is customary to follow the highest parallel of latitude which is safe for navigation.

In figure 29, *FLVMT* is the great circle joining *F* and *T*. Latitudes higher than the parallel of *LM* are assumed to be dangerous. The ship cannot, therefore, follow the great-circle arc *LVM*. Nor would she go from *F* to *L*, along to *M* and then down to *T*. The shortest route she can take is *FABT* where *FA* and *BT* are great-circle arcs tangential to the parallel at *A* and *B*.

FABT is called a *composite track*.

It is the shortest route because, if *L* and *M* are taken as any points on the parallel outside the part *AB*, $(FL+LA)$ is greater than *FA*, and $(BM+MT)$ is greater than *BT*. Also since *A* is the point nearest the pole on the great circle of which *FA* is an arc, any other great circle from *F* to a point between *A* and *B* would cut the parallel between *L* and *A* and so carry the ship into danger. (See Volume III.)

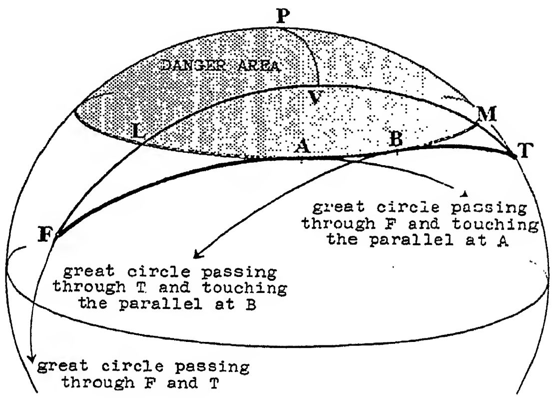


FIGURE 29.

If, in the last example, 60°S. is taken as the highest 'safe' latitude, it can be shown that the length of the composite track is 5,132′. This track, together with the great-circle and rhumb-line tracks, is drawn for comparison on polar gnomonic and Mercator charts on page 55 of Chapter V.

The Vertex. The point nearest the pole that a great circle reaches is called its *vertex*—*V* in figure 29.

At this point the great circle ceases to approach the pole and begins to draw away. It must therefore cut the meridian through the vertex at right angles.

To Find the Position of the Vertex. Napier's mnemonic rules—see page 251 of the Appendix—provide a ready means of finding the position of the vertex.

Figure 30a shows a great-circle track FT and its vertex V in relation to the pole P. Figure 30b shows the circular parts of the triangle PFV. It is assumed that the initial course F has been calculated.

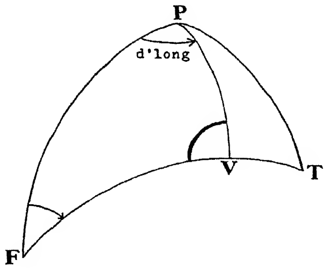


FIGURE 30a.

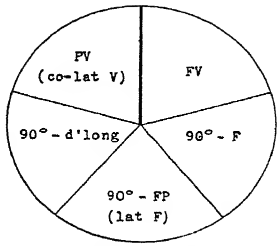


FIGURE 30b.

The quantities required are PV, which defines the latitude of V, and the angle FPV, which is the difference of longitude between F and V.

By Napier's second rule :

$$\sin PV = \cos (90° - FP) \cos (90° - F)$$

i.e. $$\cos (\text{lat. } V) = \cos (\text{lat. } F) \sin F$$

By Napier's first rule :

$$\sin (90° - FP) = \tan (90° - F) \tan (90° - \text{d'long})$$

$$\therefore \quad \cot (\text{d'long}) = \sin (\text{lat. } F) \tan F$$

In the previous example, worked on page 36, F was 43°S., 140°E.; T was 56°S., 70°W.; and the angle $P'FT$ was found to be 16°38′. The latitude of the vertex of the great circle joining F to T is therefore given by:

$$\cos (\text{lat. } V) = \cos 43° \sin 16°38'$$

i.e. $$\log \cos (\text{lat. } V) = 9{\cdot}864\ 13$$
$$+9{\cdot}456\ 74$$

$$9{\cdot}320\ 87$$

The latitude of V is thus 77°55′S.

The longitude of V is given by:

$$\cot (\text{d'long}) = \sin 43° \tan 16°38'$$

i.e. $$\log \cot (\text{d'long}) = 9{\cdot}833\ 78$$
$$+9{\cdot}475\ 30$$

$$9{\cdot}309\ 08$$

The d'long between F and V is thus 78°29′E., and the position of the vertex is:

$$\begin{cases} 77°55'\text{S.} \\ 141°31'\text{W.} \end{cases}$$

CHAPTER V

CHARTS

In the practice of navigation it is necessary that the navigator should have drawings of the Earth's surface on which to lay off his proposed course, fix the position of his ship and find where he is in relation to the land. Moreover, since convenience demands that these drawings shall be flat, his problem is to show part of the surface of a sphere, which has three dimensions, as a plane or flat surface, which has two. The sphere is not a developable figure like a cylinder or a cone which can be unrolled into a plane surface. Distortion is therefore inevitable when a flat drawing of its surface is made, and if the area covered by the drawing is large, that distortion can be considerable. It is negligible only when the area is so small that the portion of the Earth's surface which it covers may be considered plane.

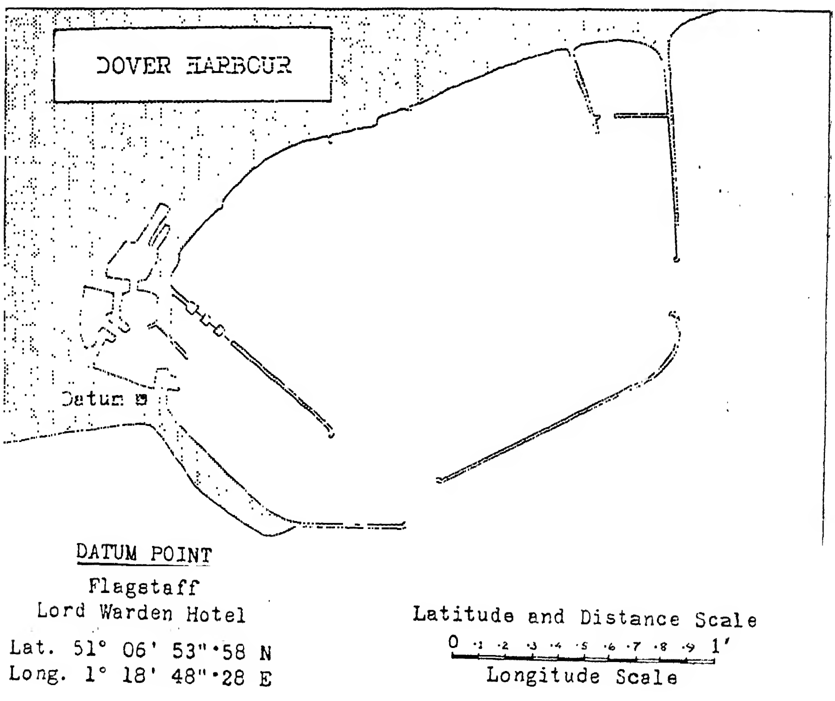


FIGURE 31.

The Plan. A plane drawing of a small portion of the Earth is called a *plan*. It is useful for showing the shape of harbours.

Two scales are sometimes given—one for longitude and one for latitude and distance—and the position of any point is fixed in relation to some datum point. (See Misc. Chart 19.) In figure 31, this point is the flagstaff at :

$$\begin{cases} 51°06'53''\cdot58\text{N.} \\ 1°18'48''\cdot28\text{E.} \end{cases}$$

If the latitude scale only is given, the longitude scale can be constructed from it by multiplying the unit of latitude by the cosine of the latitude of the area shown on the plan. For the reason given on page 44, the resulting length will be the unit of longitude.

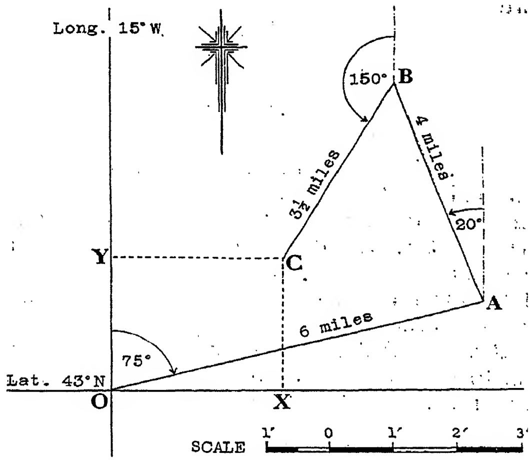


FIGURE 32.

The *natural scale* of the plan is the ratio of a length measured on the plan to the corresponding length measured on the Earth's surface.

The Plotting Chart. The navigator wishing to work out his position after manœuvring in a small area may do so by laying off distances and courses on a plan of his own making, called a *plotting chart*. On this, a convenient meridian and parallel of latitude are taken as axes, and the scale for latitude and distance is assumed to be the same everywhere on the chart. There is no separate scale for longitude. The change in longitude is found from the formula :

$$\text{d'long} = \text{departure} \times \sec\ (\text{mean latitude})$$

Figure 32 shows the track of a ship as it would appear on a plotting chart if the ship steamed 6′ at 075°, a distance and course

indicated by OA; 4′ at 340° (AB); and $3\frac{1}{2}$′ at 210° (BC). The position of C is then fixed in relation to O by its d'lat and departure.

d'lat (CX) = 2′·3N.
departure (CY) = 2′·7E.

If the position of O is 43°N., 15°W., the latitude of C is :

lat. O	43°00′·0N.
d'lat	2′·3N.
lat. C	43°02′·3N.

By calculation (2′·7 × sec 43°) or from the tables (for departure 2′·7 and latitude 43°) the d'long is seen to be 3′·7E. The longitude of C is therefore :

long. O	15°00′·0W.
d'long	3′·7E.
long. C	14°56′·3W.

Map Projections. The distortion which is inevitable when the area to be shown is large takes different forms according as different methods are adopted for showing the area, and the cartographer's object is to arrange that the distortion resulting from a particular method shall serve a particular purpose. The graticule upon which the area is delineated is formed by the meridians and parallels passing through the area, and the term *projection* is used to denote the type of graticule selected. It does not necessarily mean that the graticule is formed by the actual projection of meridians and parallels from a chosen point on to a particular plane.

THE MERCATOR CHART

To the navigator the most useful chart is one on which he can show the track of his ship by drawing a straight line between his starting point and his destination, and so be able to measure the steady course he must steer in order to arrive there. This the Mercator chart (first published in 1569) permits him to do because it is constructed so that :

(1) rhumb lines on the Earth appear as straight lines on the chart.

(2) the angles between these rhumb lines are unaltered.

It therefore follows that :

(1) the equator, which is a rhumb line as well as a great circle, appears on the chart as a straight line.

(2) the parallels of latitude appear as straight lines parallel to the equator.

(3) the meridians appear as straight lines perpendicular to the equator.

NOTE. Although a meridian is shown on the chart as a straight line and is therefore, by the principle of the chart's construction, a rhumb line over the length that appears on the chart, the definition of a meridian as a rhumb line breaks down at the pole. If there were not this mathematical discontinuity at the pole, it would be possible to steer a rhumb-line course between two places, F and T, differing by 180° in longitude, by proceeding up the meridian FP and then down the meridian PT. But the course along FP is *north*, whereas the course along PT is *south*. At P there has been an alteration of 180°. The course from F to T is not therefore the same at all points, and FPT is not a rhumb-line track. This fact, however, is important only in the exact definition of the relation between a meridian and a rhumb line. It has no practical significance because the pole can never be represented on a Mercator chart. In practice, a meridian is a rhumb line.

In the general classification of graticules adopted by cartographers, the Mercator chart is described as a 'cylindrical orthomorphic projection'. The reason for this is explained fully in Volume III. The graticule, however, is *not* a true or perspective projection of the meridians and parallels formed by drawing straight lines from the centre of the sphere through certain points on the sphere and noting the corresponding points on a circumscribing

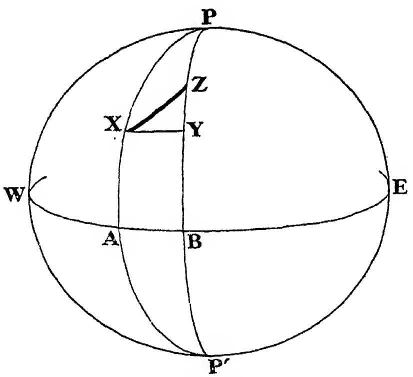


FIGURE 33a.

FIGURE 33b.

cylinder: it is a graticule made according to mathematical laws, chosen because they give the chart the properties described.

Scale on the Mercator Chart. Since the equator is shown on the chart as a straight line of definite length, and the meridians appear as straight lines perpendicular to it, the longitude scale is fixed by that length and is constant in all latitudes.

Figure 33a shows the Earth, and figure 33b the Mercator chart.

XY is part of any parallel of latitude between the meridians PAP' and PBP'. On the chart this is shown as xy. Then :

$$AB = ab = xy$$

But :

$$AB = XY \text{ sec (latitude of } X)$$

Therefore :

$$xy = XY \text{ sec (latitude of } X)$$

If the triangle XYZ is formed by rhumb lines and is small enough to be considered plane, then, since the angles are correctly shown on the chart, the triangle XYZ is similar to the triangle xyz, and the ratios yz/YZ and xz/XZ are each equal to the ratio xy/XY. Hence, for all short distances near X, the scale of the chart in the neighbourhood of x must be the scale of xy ; that is, the scale at the equator multiplied by the secant of the latitude of X.

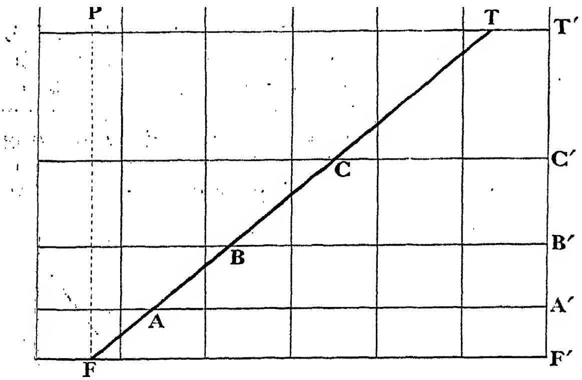


FIGURE 34.

For this reason the amount of distortion in any latitude is governed by the secant of that latitude. Greenland, in 70°N., for example, appears as broad as Africa at the equator, although Africa is three times as broad as Greenland, because the secant of 70° is 3. For a similar reason Borneo, an island on the equator, appears about the same size as Iceland in 65°N., although in area Borneo is about five and a half times as large as Iceland.

Graduation of Charts. Mercator charts are graduated along the left and right-hand edges for latitude and distance, and along the top and bottom for longitude.

The longitude scale is used only for laying down or taking off the longitude of a place, never for measuring a distance.

Measurement of Distances on the Chart. As explained in Chapter III, the length of the rhumb line between two places is referred to as the distance between them.

In figure 34, $FABCT$ is a rhumb line as it appears on the chart. AA', BB' . . . are parallels of latitude.

The distance FA must be measured on the scale between F' and A', the distance AB on the scale between A' and B', and so on. If FT is not large—less than 100′, that is—no appreciable error is made by measuring it on the scale roughly abreast of its middle point.

Meridional Parts. Since the latitude and distance scale at any part of a Mercator chart is proportional to the secant of the latitude of that part, this scale continually increases as it recedes from the equator, until, at the pole, it becomes infinite. (For this reason the complete polar regions cannot be shown on a Mercator chart.) It therefore affords no means of comparison with the fixed longitude scale. The tangent of the course-angle PFT, for example, is not PT divided by FP, where PT is measured on the longitude scale and FP on the latitude scale. For that ratio to be valid, PT and FP must be measured in the same fixed units. The fixed longitude scale provides this unit, which is the length of one minute of arc on that scale. This length is called a *meridional part,* and gives rise to the definition :

The meridional parts of any latitude are the number of longitude units in the length of a meridian between the parallel of that latitude and the equator.

If the longitude scale on the Mercator chart is 1 degree or 60 meridional parts to an inch, the length of the meridian between the parallel of 45°N. and the equator, when measured on the chart, is not 45 inches but 50·5 inches, the length of 3,029·9 meridional parts as found from *Inman's Tables* for latitude 45°. Meridional parts thus involve chart-lengths. They are not in any way concerned with distance on the Earth's surface. That is expressed in nautical miles.

To Find the Meridional Parts of any Latitude. In figure 35, the upper half of which shows a part of the Earth, F is a point on the equator, and FT the rhumb line joining it to T. The lower half shows this same rhumb line as the straight line ft on a Mercator chart.

If TQ is now divided into n small lengths, α, so that (n.α) is equal to the latitude of T, the arcs of parallels drawn through the points of division are equally spaced and, with the meridians, form a series of small triangles FAX, ABY . . . If, furthermore, α is so small that these triangles may be considered plane, they are equal in all respects since :

$$FX=AY= \ldots =\alpha$$
$$\angle X=\angle Y= \ldots =\text{one right angle}$$
$$\angle F=\angle A= \ldots =\text{the course}$$

Therefore $AX=BY= \ldots$, and, since these small arcs recede in succession from the equator, the meridians which bound them are spaced successively farther apart. Hence :

$$FQ_1<Q_1Q_2< \ldots$$

A comparison of the two halves of the figure should make clear the relation between the small triangles as they are when drawn on

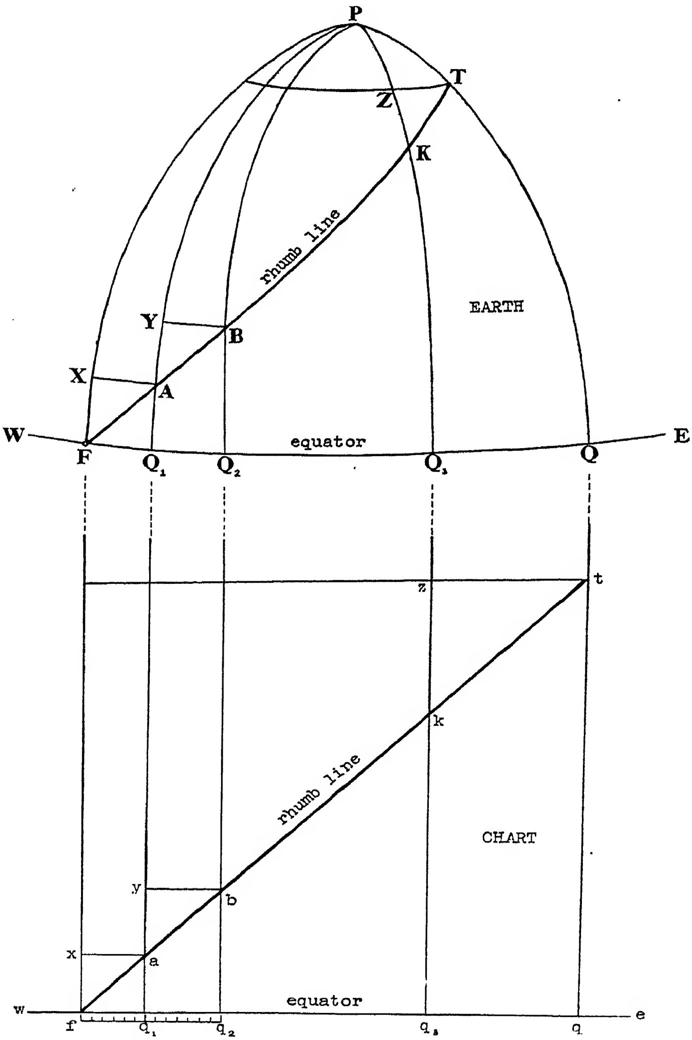


FIGURE 35.

the Earth and as they appear when drawn on the chart. On the Earth they are all equal, but on the chart they are only similar.

They increase progressively as they recede from the equator. This increase can be found by considering two similar and corresponding triangles. Thus :

$$\frac{fx}{FX}=\frac{ax}{AX}=\frac{FQ_1}{AX}=\text{sec (latitude } A)$$

$$fx=FX \text{ sec (latitude } A)$$

$$=\alpha \sec \alpha$$

Similarly, by considering the triangles ABY and aby :

$$ay=\alpha \sec 2\alpha$$

But qt, the length of the meridian between the parallel through t and the equator, is the sum of all the elements fx, ay . . . kz. That is :

$$qt=\alpha(\sec \alpha+\sec 2\alpha+\sec 3\alpha+ \ . \ . \ . \ +\sec n\alpha)$$

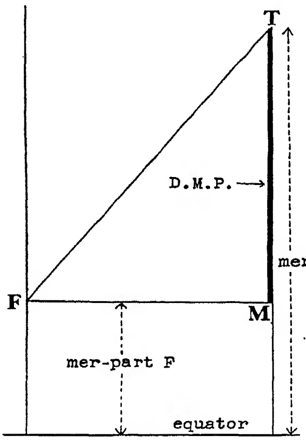


FIGURE 36a.

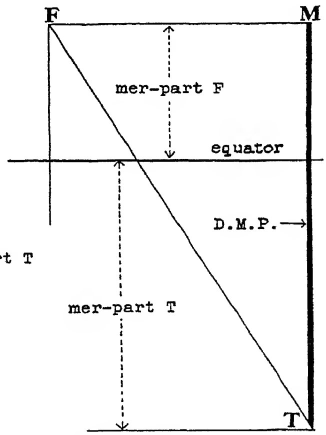


FIGURE 36b.

The expression thus gives the number of meridional parts in the latitude of T, and *Inman's Tables* include a table giving the values it assumes for every degree and minute of latitude.

NOTE. The reader who is acquainted with the Integral Calculus will recognise that this expression for finding the meridional parts of latitude l is simply a variant of the integral form :

$$\int_0^l \sec l \, dl$$

—the value of which is :

$$\log_e \cot\left(\frac{\pi}{4}-\frac{l}{2}\right)$$

—where l is in radians. This method of approach is discussed in Volume III.

Difference of Meridional Parts. (D.M.P.) When F does not lie on the equator, the latitude of F has its own meridional parts, a length referred to as 'mer-part F'.

The number of meridional parts in the length of a meridian between the parallels through F and T will therefore be:

(1) F *and* T *on the same side of the equator* (*Figure* 36*a*)
mer-part T *minus* mer-part F.

(2) F *and* T *on opposite sides of the equator* (*Figure* 36*b*)
mer-part T *plus* mer-part F.

This length MT is always called the *difference of meridional parts* and written D.M.P.

MERCATOR SAILING

To Find the Course by Mercator Sailing. From the triangle FTM in figures 36a and 36b, it is apparent that:

$$\tan\,(\text{course}) = \frac{FM}{MT} = \frac{\text{d'long}}{\text{D.M.P}}$$

The angle thus obtained is exact, no matter what the length of FT is. That length, as in plane sailing, is obtained from the formula:

$$\text{distance} = \text{d'lat sec (course)}$$

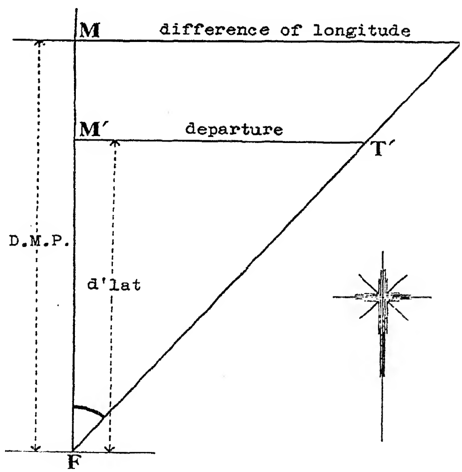


FIGURE 37.

Figure 37 shows the relation between the two methods of finding the course. In the mercatorial method the d'lat is stretched into D.M.P. and the d'long remains unchanged; in the other, the d'lat

remains unchanged and the d'long is compressed into departure. Hence :

$$\frac{\text{d'long}}{\quad} = \tan \qquad \frac{\text{dep.}}{\quad}$$

The use of the departure formula, however, involves the labour of finding an accurate middle latitude if an error in the course is to be avoided. For this reason, the mercatorial formula is preferable.

Suppose, for example, that F is 16°00′S., 5°55′W., and that T is 40°28′N., 74°00′W.

To obtain the course and distance between them, proceed thus :

lat. F	16°00′S.	mer-part F	972′·73S.	long. F	5°55′W.
lat. T	40°28′N.	mer-part T	2,659′·37N.	long. T	74°00′W.
d'lat	56°28′N.	D.M.P.	3,632′·10N.	d'long	68°05′W.
	60				60
	3,388′N.				4,035′W

tan (course) = 4,085·0

log 4,085·0	3·611	19
log 3,632·1	3·560	16
log tan (course)	0·051	03

i.e. course = N.48°21′·5W. (true)
311½°

distance = d'lat sec (course)
= 3,388′ sec (48°21′·5)

log 3,388	3·529	94
log sec (48°21′·5)	0·177	52
log distance	3·707	46

∴ distance = 5,099′

Again, suppose a ship leaves a position 50°N., 17°W., and steams for 1,200′ on a course of 260°. What is her position at the end of this run ?

The formula used now is :

$$\text{d'lat} = \text{distance} \cos (\text{course})$$
$$= 1{,}200 \cos 80°$$

—since a course of 260° is equal to S.80°W. (true). Then :

log 1,200	3·079	18
log cos 80°	9·239	67
log d'lat	2·318	85

The d'lat is therefore 208'·4S., and from it the position of T is obtained. Thus :

lat. F	50°00'·0N.	mer-part F	3,474'·47
d'lat	3°28'·4S.	mer-part T	3,161'·26
lat. T	46°31'·6N.	D.M.P.	313'·21

$$\text{d'long} = \text{D.M.P. tan (course)}$$
$$= 313'{\cdot}21 \tan 80°$$

log 313'·21	2·495 84	long. F	17°00'·0W.
log tan 80°	0·753 68	d'long	29°36'·3W.
log d'long	3·249 52	long. T	46°36'·3W.
d'long	1,776'·3W.		

The position of T is therefore $\begin{cases} 46°31'{\cdot}6\text{N.} \\ 46°36'{\cdot}3\text{W.} \end{cases}$

To Construct a Mercator Chart. Since there is no distortion at the equator, the base on which the chart is built must be the line representing the equator, and convenience governs the length of this line. Suppose it is 36 inches. Then the longitude scale must be :

$$\frac{\text{length of equator in degrees}}{\text{length of base in inches}} = \frac{360 \text{ degrees}}{36 \text{ inches}}$$

—that is, 10° of longitude or 600 meridional parts to one inch. Vertically the scale will be the same—600 meridional parts to one inch.

If it is required to draw the meridians for every 20°, say, the equatorial line must be divided into eighteen equal parts, two inches long. The perpendiculars drawn through the points of division will be the meridians.

The one through the left-hand extremity will be the meridian of 180°W., and the one through the right-hand extremity the meridian of 180°E.

The table of meridional parts gives all the information necessary for deciding the position of the parallels of latitude.

The number of meridional parts between the parallel of 20° and the equator is 1,225·14, and since these are shown on a scale of 600 to one inch, the parallels of 20° must be drawn (1,225·14÷600) or 2·04 inches from the equatorial line on the chart.

The number of meridional parts between the parallel of 40° and the equator is 2,622·69. The parallels of 40° are therefore drawn (2,622·69÷600) or 4·37 inches from the equatorial line.

In the same way the other parallels are drawn, and on the graticule thus formed it is possible to insert the position of any place the latitude and longitude of which are known.

To Construct a Mercator Chart on a Large Scale. In order that small portions of the Earth may be shown in detail, it is necessary to employ a large scale and construct only the relevant portion of the chart. If it so happens that the equator is not included, the chart-lengths between successive parallels of latitude on the chart are found by reducing to inches, according to the scale employed, the difference between the corresponding meridional parts.

Suppose, for example, it is required to construct a chart from 142°E. to 146°E., and 54°N. to 58°N., the scale of the chart being 1° of longitude to 1 inch, or 1′ of longitude to 1/60 inch.

The difference of longitude between the outer meridians to be shown is 4°, and, since the scale of the chart is 1° of longitude to 1 inch, the line at the bottom of the chart representing the parallel of 54°N. is 4 inches long.

The meridians of 142°, 143°, 144°, 145°, and 146° will be the perpendiculars erected on this line at its two ends and at the points dividing it into four equal parts.

The lengths in inches between the parallels of 54° to 58° can be taken direct from *Inman's Tables* where the figures are given for a chart constructed on a scale of one inch to each degree of longitude; or they can be found from the table of meridional parts.

Latitude	Meridional Parts	D.M.P.	Chart-length between Parallels
58°	4,294·30		
		111·68	1·86 inches
57°	4,182·62		
		108·72	1·81 ,,
56°	4,073·90		
		105·93	1·76 ,,
55°	3,967·97		
		103·33	1·72 ,,
54°	3,864·64		

In order to increase the accuracy with which positions can be plotted, the chart-lengths between meridians and between parallels are divided, if necessary, into convenient units—10′ of longitude between meridians, and 10′ of latitude between parallels in figure 38. This division is easily effected on the longitude scale because that is fixed. On the latitude scale, however, it can be carried out only with the further aid of the table of meridional parts which is now entered for every 10′ between 54° and 58° instead of every degree.

Figure 38 shows the complete graticule. Each rectangle, whatever its dimensions in inches, represents a part of the Earth's surface bounded by meridians 1° apart in longitude, and parallels 1° apart in latitude; and, although the chart-lengths between these parallels vary from 1·72 inches to 1·86 inches as shown, each length represents a distance of 60 miles on the Earth's surface. The actual

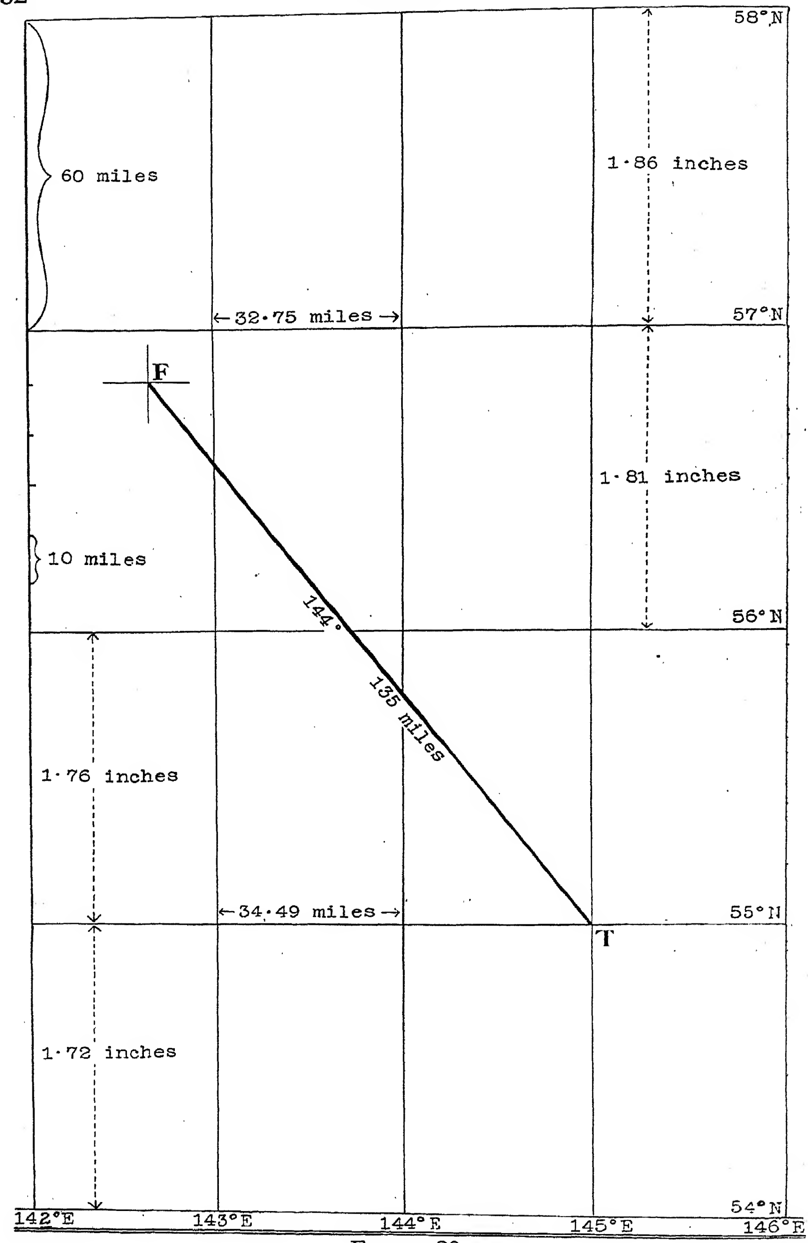

58° N
57° N
56° N
55° N
54° N
60 miles
10 miles
1·86 inches
1·81 inches
1·76 inches
1·72 inches
←32·75 miles→
←34·49 miles→
F
T
144°
135 miles
142° E
143° E
144° E
145° E
146° E

FIGURE 38.

distance in miles between the meridians depends on the latitude in which it is measured.

As already explained on page 45, distances between places must be measured on the latitude scale abreast of the places. The distance between F and T, for example, is measured on the latitude scale between 55° and 57°, and is found to be 135 miles.

Great-Circle Tracks on a Mercator Chart. Since only rhumb lines appear as straight lines on a Mercator chart, great circles will in general appear as curves.

Also, since the limiting great circles are the equator, which appears as a horizontal line, and any double meridian, which appears

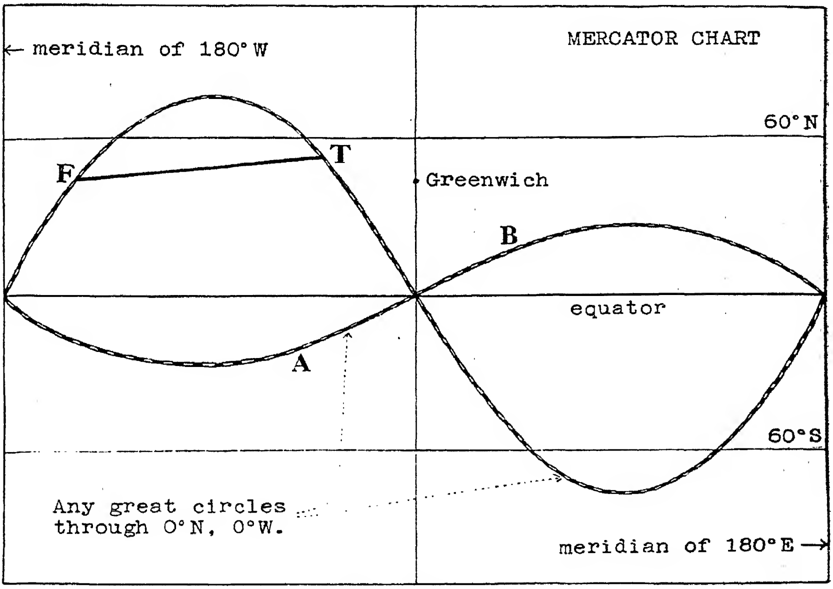


FIGURE 39.

as two separate lines 180° apart and perpendicular to it, any great circle passing through their points of intersection must curve towards the poles, as shown in figure 39. The great circle joining F and T will always lie, therefore, on the polar side of the rhumb line joining them, and when the difference of latitude between F and T is small and the difference of longitude large, it is seen that the difference between the two tracks is considerable. If, however, the two points lie on opposite sides of the equator, as at A and B, then the rhumb line almost coincides with the great circle. (This is the reason for the note to the example on great-circle sailing on page 35.)

GREAT-CIRCLE OR GNOMONIC CHARTS

In order to assist the navigator in finding the great-circle track between two places, charts are constructed so that any straight line drawn on them shall represent a great circle. These are known as *gnomonic charts,* and they are formed by projecting the Earth's surface from the Earth's centre on to the tangent plane at any convenient point.

Since a great circle is formed by the section of a plane through the Earth's centre with the Earth's surface, and one plane cuts another in a straight line, all great circles will appear on the chart as

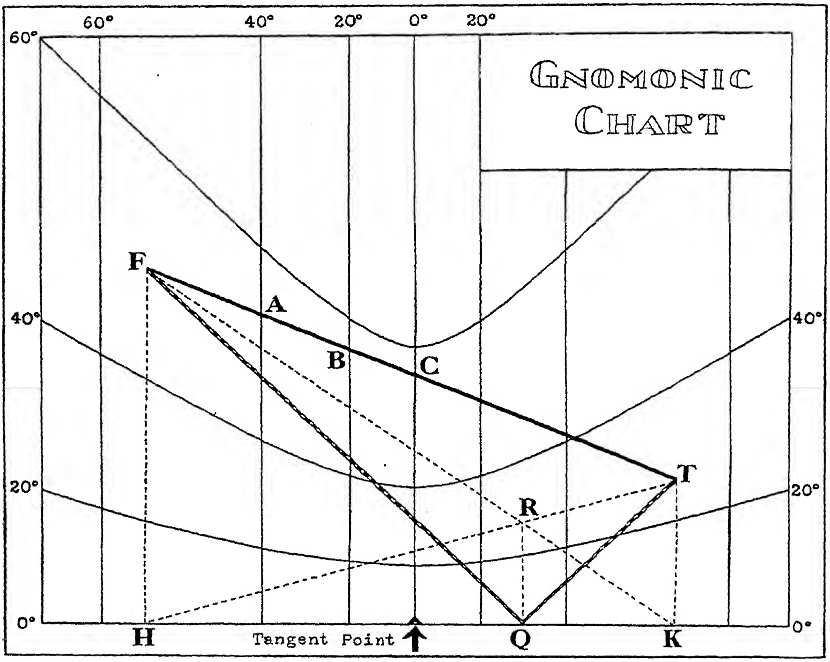


FIGURE 40.

straight lines. But the meridians will not be parallel unless the tangent point is on the equator. Nor will rhumb lines be straight. Also angles are distorted except at the tangent point. It is therefore impossible to take courses and distances from a gnomonic chart.

The mathematical theory of this chart is explained in Volume III.

Figure 40 shows the graticule of a gnomonic chart in which the tangent point is on the equator, and it will be noticed that the graticule is symmetrical about the meridian through this tangent point, which is independent of the longitude. The longitude scale

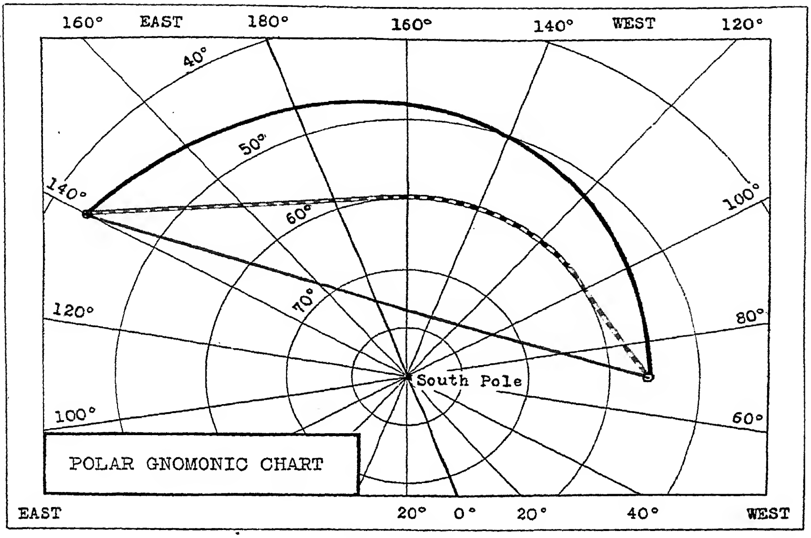
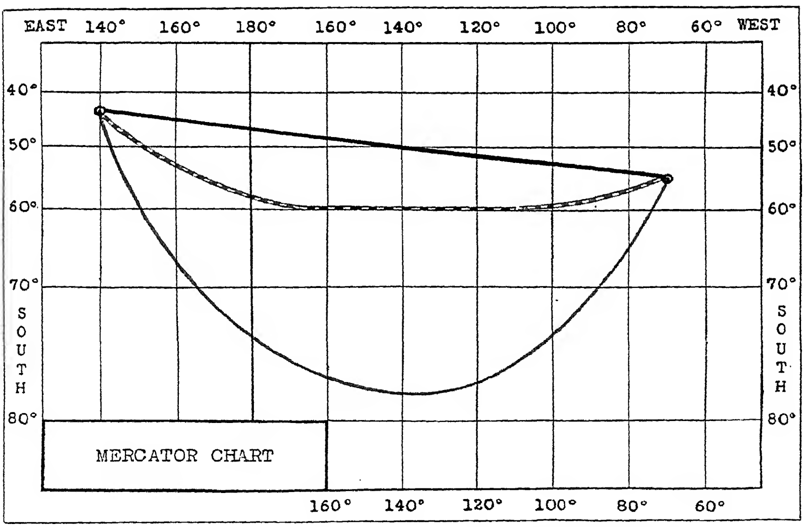


FIGURE 41a.



Great Circle Track
Rhumb Line ...
Composite ...

FIGURE 41b.

can therefore be adjusted to suit the navigator's convenience. In the figure the tangent point is in longitude 0°.

The *Meade Great-Circle Diagram* is a graticule of this type.

To Transfer a Great-Circle Track to a Mercator Chart. The transference of a great-circle track, such as FT in figure 40, from a gnomonic to a Mercator chart, which is the navigational one, is done by noting the latitude and longitude of convenient points A, B, C . . . on the line FT, marking these points on the Mercator chart, and joining them by a smooth curve.

When F and T lie on opposite sides of the equator, F being north and T south, the same chart can be used because a gnomonic chart of both hemispheres when the tangent point is on the equator must be symmetrical about the equator. The following geometrical construction therefore suffices:

(1) Mark the position of T as if it were in the northern hemisphere.

(2) Join F to K, the point on the equator which has T's longitude.

(3) Join T to H, the point on the equator which has F's longitude.

(4) Drop a perpendicular RQ on the equator from R, the point where FK cuts TH.

(5) Draw FQ and QT. Then FQ is the great-circle track in the northern hemisphere, and QT is the reflexion of its continuation south of the equator. Points on QT may therefore be treated as if they were in the southern hemisphere.

The Polar Gnomonic Chart. This is a gnomonic chart in which the pole is a tangent point. The meridians therefore radiate from the pole and parallels of latitude appear as concentric circles. It is thus an easy chart to read.

Figure 41a shows a polar gnomonic chart for the south polar regions on which are drawn the three tracks—rhumb line, great circle and composite—between Tasmania and Cape Horn referred to on pages 36 and 38.

For comparison, the same tracks are shown (figure 41b) on a Mercator chart of the same area.

CHAPTER VI

THE SHIP'S POSITION

The ship's position can be found by calculation based on the last known position or by reference to any convenient objects, terrestrial or heavenly. This reference commonly takes the form of direct observation.

POSITION BY CALCULATION

Dead Reckoning. (D.R.) This expression, a corruption of the old '*ded*uced reckoning' or 'position by account', is used to cover all positions that are obtained from the course the ship steers and her speed through the water, and from no other factors. The D.R. position is therefore an approximate position. It is also the position recorded by an automatic plotting machine. (This machine is controlled by the ship's gyro-compass and the ordinary log that gives the ship's speed through the water.)

Estimated Position. (E.P.) This position is the best that the navigator can obtain by calculation. It is the position arrived at when the D.R. position is adjusted for the estimated effects of wind, currents and tidal streams, but it is still an approximate position because the exact influence on the ship's course and speed of variable factors, such as wind and movement of the water, cannot be assessed.

Detailed information about currents and tidal streams may be obtained from Admiralty charts, sailing directions, tidal publications, tidal atlases and current charts; and the direction of a current or tidal stream is always given as that in which the water is moving. A current is said to *set* 150° at 2 knots, and a ship that experiences it for three hours, say, is said to be set six miles in a direction 150°.

The system of naming the natural movements of the ocean is exactly opposite to the system of naming those of the winds. A northerly wind, for example, blows *from* the north, but a northerly current sets *to* the north.

Dead Reckoning by Traverse Table. The use of a traverse table for giving the ship's position at the end of a straight run is explained on page 25. If the ship alters course from time to time, the traverse table can still be used to give her final position.

Suppose, for example, that a ship, in position 50°14′N., 16°11′W., at 0800 and steaming 132° at 15 knots, makes the following alterations of course :

(1) at 0840, new course 246°
(2) at 0956 ,, 302°
(3) at 1032 ,, 010°
(4) at 1144 ,, 090°

What is her D.R. position at noon ?

Time	Interval in Minutes	True Course	Distance Run	D'lat N.	D'lat S.	Departure E.	Departure W.
0800–0840	40	S.48°E.	10′	—	6′·7	7′·4	—
0840–0956	76	S.66°W.	19′	—	7′·7	—	17′·4
0956–1032	36	N.58°W.	9′	4′·8	—	—	7′·6
1032–1144	72	N.10°E.	18′	17′·7	—	3′·1	—
1144–1200	16	E.	4′	—	—	4′·0	—
				22′·5 14′·4	14′·4	14′·5	25′·0 14′·5
				d'lat 8′·1N.		dep. 10′·5W.	

lat. *F*	50°14′·0N.	mean lat. 50°18′N.	long. *F*	16°11′·0W.
d'lat	8′·1N.		d'long	16′·4W.
lat. *T*	50°22′·1N.		long. *T*	16°27′·4W.

D.R. position at noon 50°22′·1N. 16°27′·4W.

Estimated Position by Traverse Table. If, in the previous example, the navigator estimated that the tidal stream experienced had set the ship 4′ in a direction 062°, an extra line in the working of the traverse would be necessary. The set is treated as another course and distance, and the line would read :

True Course	Distance	D'lat	Departure
N.62°E.	4′		3′·5E.

The total d'lat is now 10′N., the total departure 7′W., and the d'long 11′W.

The estimated position at noon is therefore $\begin{cases} 50°24'\text{N.} \\ 16°22'\text{W.} \end{cases}$

Plotting the Ship's Track. The procedure for plotting the ship's track on a simple plotting chart is explained on page 41. When it is plotted on a Mercator chart, courses are laid off from the compass rose by means of parallel rulers ; distances are taken from the

appropriate part of the latitude scale with dividers ; and the latitude and longitude of the D.R. or E.P. are noted at once.

The methods of using parallel rulers and dividers are fully explained in Volume I.

To Keep the Reckoning in Tidal Waters. A ship moving in a tidal stream has two velocities : one relative to the water through which she moves at whatever speed is imparted to her by her propellers, and the other imparted by the water itself which carries her in the direction in which it happens to be flowing and at the speed of its flow. These two velocities combine to give her a course and speed *made good over the ground.*

Suppose the ship is steering in a direction FA through a tidal stream (or current) setting 090°. (Figure 42.)

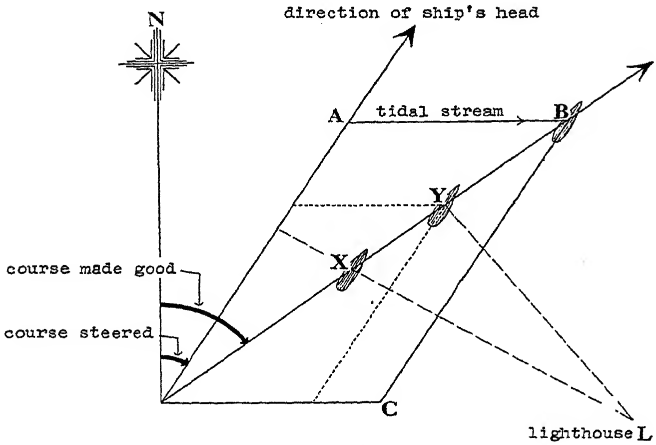


FIGURE 42.

At the end of one hour, say, she would have reached A, had there been no stream. But in that time the stream has carried her a distance AB to the east, and at the end of the hour she finds herself at B. Over the ground she therefore makes good a course indicated by NFB and a speed by FB.

In effect, the ship proceeds crab-fashion, and it is important to remember this. For example, the lighthouse in figure 42 is on the beam, not when the ship is at Y, where LY is perpendicular to FB, but when she is at X, where LX is perpendicular to FA.

To Shape a Course in Tidal Waters. When the navigator knows the direction of the place he wishes to reach and the current or tidal stream he will experience on passage, his problem is to find the course to steer. In other words, given AB and the direction of FB, how may FA be found ?

If, in figure 43, FT is the direction in which he wishes to move over the ground, and the tidal stream is again setting 090°, then it is clear that the course he must steer in order to make good a direction FT must lie to the left of FT in order that the stream may carry the ship on to FT.

Let FC represent the set of the stream in one hour. With centre C and radius equal to the distance the ship steams in one hour, draw an arc of a circle cutting FT in B. Then CB is the direction in which the ship must steer.

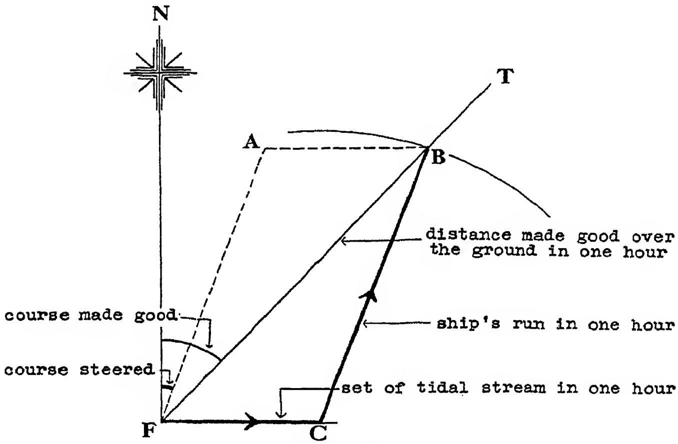


FIGURE 43.

If the parallelogram $FABC$ is completed by drawing FA parallel to CB and AB parallel to FC, it is seen that, in principle, figure 43 does not differ from figure 42.

Should it be necessary to make good a certain speed in the direction FT, then FB must be drawn to represent this speed, and B must be joined to C. CB then indicates the direction in which to steer and the speed at which to steam.

POSITION BY OBSERVATION

The Position Line. In order that the navigator may be sure that he is following the course he has decided upon, it is necessary for him to ascertain his position from time to time. This he does by observing terrestrial or heavenly bodies and from his observations finding what are known as position lines.

A position line is any line, drawn on the chart, on which the ship's position is known to lie.

It is usually a straight line.

The Line of Bearing. The simplest form of position line is the line of bearing obtained from a terrestrial object, the position of which is known.

Suppose, for example, a lighthouse is seen to bear 065°. On the chart this lighthouse is marked by the point *L*. (Figure 44.) Then a line drawn through *L* in the direction of the reverse bearing, 245°, is the position line because from no point except one that lies on *LQ* will the lighthouse bear 065°.

In order to distinguish a position line from other lines on the chart, it is always marked with an arrow head and the time at which the observation was taken is shown against it.

The Fix. If two lines of bearing can be obtained at approximately the same moment, the position of the ship must lie at their point of intersection, which is the only point common to the two lines. A position obtained thus is called a *fix*, and it is important to remember that the term *fix* is applied only to a position obtained by observations of *terrestrial* objects.

The Running Fix. If there is an appreciable interval between the observations, allowance must be made for the distance the ship

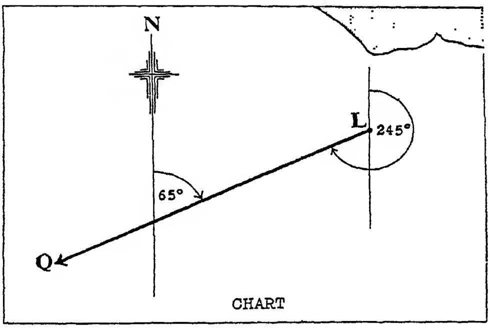


FIGURE 44.

has run in that time, and for any tidal stream or current experienced. A position obtained thus is called a *running fix*.

The Transferred Position Line. In order to obtain a running fix, the first position line must be transferred.

Suppose the first bearing is taken at 1035, and the second at 1115, as shown in figure 45. All that is known is that the ship is somewhere on the first line of bearing at 1035. Let her position be *A*. In the 40 minutes which elapse before the second bearing is taken, the ship would, in still water, reach a position *B*. But in that period the tidal stream carries her a distance *BC*. At 1115 she must therefore consider herself on a line *FC* drawn parallel to the first position line *DA*. This second line, *FC*, is known as a *transferred position line*, and a double arrow is used to distinguish it from other lines.

It will be noticed that the same running fix is obtained whatever starting point is taken on the first line of bearing. If *D*, for example,

had been taken instead of A, the transferred position line would have been drawn through F and would have coincided with the one drawn through C because DE is equal and parallel to AB, EF to BC, and therefore CF to AD.

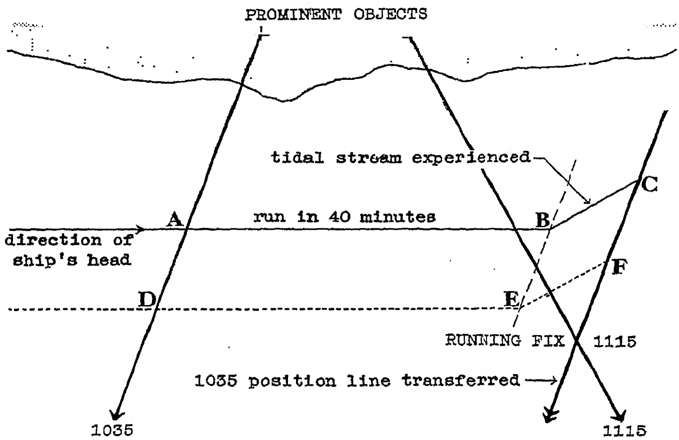


FIGURE 45.

The methods by which position lines can be obtained from terrestrial objects are discussed fully in Volume I. What has been said here is intended to show the principle governing the use

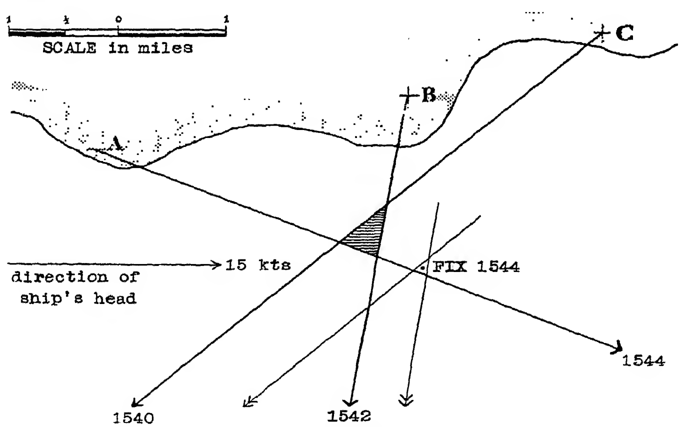


FIGURE 46.

of position lines, because that remains unchanged whether the lines are obtained from terrestrial objects or heavenly bodies. The observation and subsequent calculation which together form what

is known as a sight do no more than give the navigator a single position line.

The Observed Position. (OBS.) It is convenient to distinguish between the position obtained by observations of terrestrial objects and that by observations of heavenly bodies, because the latter is not so reliable. For this reason the position decided by the point of intersection of two position lines derived from astronomical observations, or deduced from a number of such position lines, is known as an *observed position*.

The Cocked Hat. When three position lines are obtained within a few minutes, they will not, as a rule, meet in a point : they will form the shaded triangle shown in figure 46. This triangle is known as a *cocked hat*.

In this example, there are two-minute runs at 15 knots between the times at which the lines of bearing are obtained. The line of bearing through *C*, obtained at 1540, must therefore be transferred 1 mile in the direction of the ship's course, and that through *B*, taken at 1542, half a mile. When these adjustments are made, the cocked hat disappears almost entirely.

If the cocked hat does not disappear, but is reasonably small, the position of the ship is usually taken to be the middle of it.

If the cocked hat is not small, and there is no reason for suspecting that one bearing is not to be trusted, the navigator would take his position to be that vertex which would place him closest to danger.

The mathematical problem of the cocked hat is explained fully in Volume III.

CHAPTER VII

THE CELESTIAL SPHERE

To an observer on the Earth, the sky has the appearance of an inverted bowl, so that the stars and other heavenly bodies, irrespectively of their actual distances from the Earth, appear to be situated on the inside of a sphere of immense radius, described about the Earth as centre. This is called the *celestial sphere.*

The Celestial Poles. These are the points—P and P' in figure 47—in which the Earth's axis, if produced, would cut the celestial sphere.

The Celestial Equator. This is the great circle in which the plane of the Earth's equator cuts the celestial sphere.

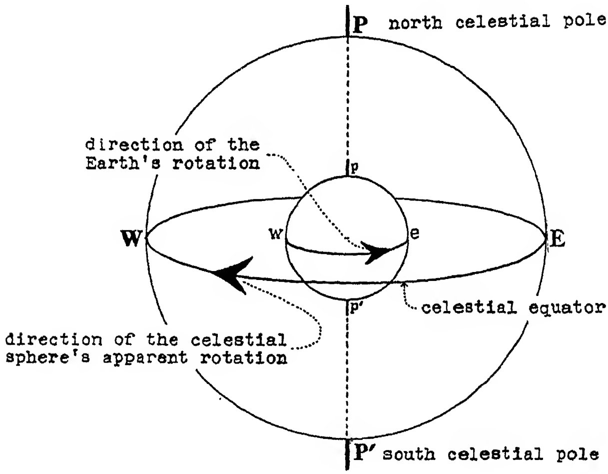


FIGURE 47.

Apparent Motion of the Celestial Sphere. Within the celestial sphere, which is fixed, the Earth rotates about its axis, turning eastward, but an observer on the Earth is not aware of this rotation unless he watches the movement of objects in no way connected with the Earth, just as a passenger in a railway train is not aware of its forward movement unless he watches the neighbouring countryside slipping backward across the carriage window. The celestial sphere therefore *appears* to rotate westward, and for this reason the Sun and the stars *appear* to rise east of and to set west of the observer's meridian.

Angular Distance between the Stars. The appearance of the stars on the celestial sphere conveys no idea of their actual distances from the Earth.

The star A, for example, in figure 48, may be ten times more remote than the star B, but the angle which the two subtend at C, the Earth's centre, is the same whether the stars are assumed to be at A' and B' or A'' and B''. To an observer on the Earth the distance between them (or between any other heavenly bodies) is thus an angular distance, and he can deal with all problems involving the measurement of that distance by the methods of spherical trigonometry.

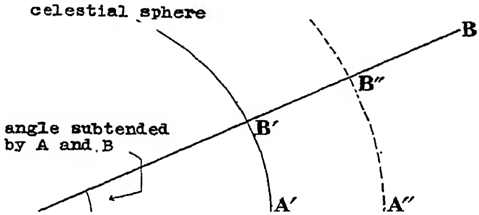


FIGURE 48.

The actual distances, moreover, are so great that any movement which the stars have in space is lost to the casual observer on the Earth. Within ordinary limits of time the angle ACB thus remains constant except for the small variation which results from the Earth's orbital motion.

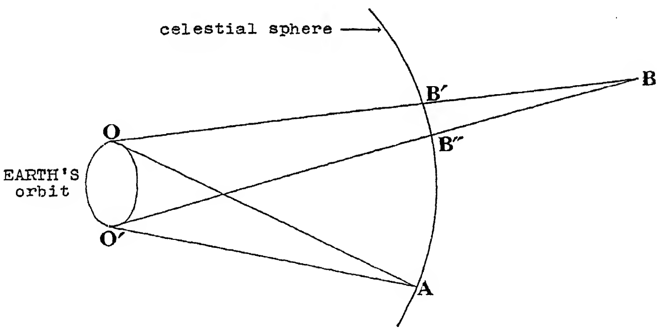


FIGURE 49.

Figure 49, which is much exaggerated for the sake of clarity, shows how this variation can occur.

A and B are two stars, the position of A being considered fixed in the celestial sphere. When the Earth is at O, the angle subtended by these stars is AOB, and B appears on the celestial sphere at B' ;

but when the Earth is at O', the other extremity of its orbit, the angle subtended is $AO'B$, and B appears on the celestial sphere at B''. The position of B therefore varies in relation to the position of A. This variation, however, is usually negligible because the length OO' is negligible in comparison with the distances of most stars.

Apparent Path of the Sun in the Celestial Sphere. When a heavenly body is actually close to the Earth, like the Sun, its position in the celestial sphere changes considerably during a year relative to the Earth and a distant star lying in the plane of the Earth's orbit. The choice of such a star gives a fixed direction in space from which angles can be measured. In the practice of astronomy, however, it is convenient to choose not an actual star but the First Point of Aries (see page 68) and this is the point represented by A in figure 50.

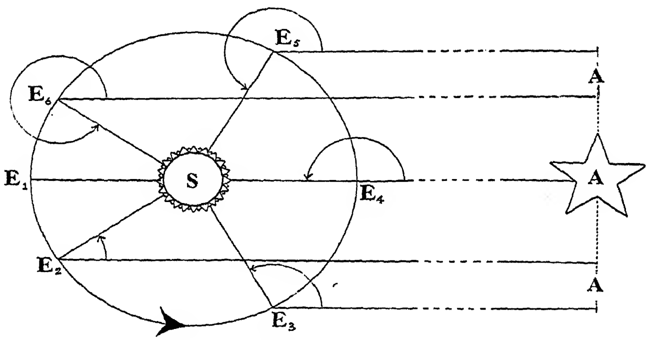


FIGURE 50.

On the 21st March the Earth is at E_1, in line with the Sun and the First Point of Aries.

A few weeks later, when the Earth has moved along its orbit to E_2, the angle subtended at the Earth by the Sun and the First Point of Aries is AE_2S, measured anti-clockwise.

When the Earth is at E_3, this angle has increased to AE_3S, and the increase continues steadily as the Earth moves through the positions E_4, E_5 and E_6, until, when the Earth is again at E_1, the angle reaches 360°. The motion is then repeated.

To an observer on the Earth this motion is revealed as a change in the Sun's position in the celestial sphere.

When the Earth is at E_1 (figure 50) A and S appear to coincide in the celestial sphere at a point S_1 in figure 51.

When the Earth has moved to E_2 (figure 50) the Sun, to this observer, appears at S_2, where the angles AE_2S (figure 50) and AES_2 (figure 51) are equal.

The Sun therefore passes through positions S_1, S_2 . . . in the celestial sphere, corresponding to the positions E_1, E_2 . . . actually passed through by the Earth during its orbit round the Sun ; and since the angle itself increases from 0° to 360° during the year, the Sun, to an observer on the Earth, must appear to describe one complete circle in the celestial sphere during the same period.

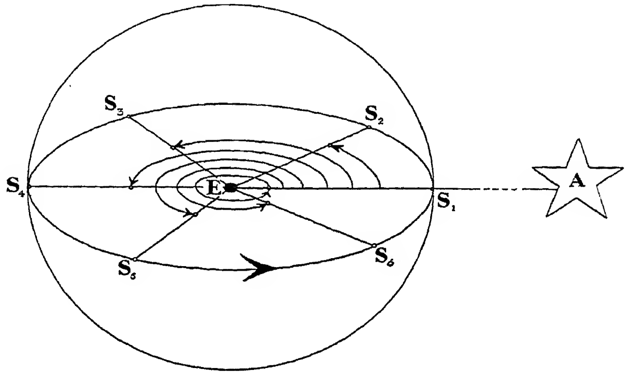


FIGURE 51.

The Ecliptic. This apparent path of the Sun in the celestial sphere is called the *ecliptic*. It is a great circle, and it makes an angle of 23°27′ with the celestial equator because the Earth's axis of rotation is tilted that amount from the perpendicular to the plane of the Earth's orbit.

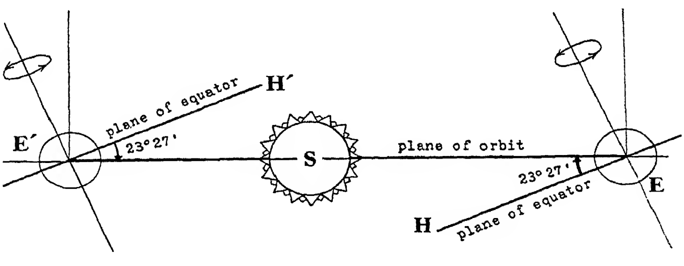


FIGURE 52.

Figure 52 shows the tilt of the Earth's axis in relation to the Sun when the Earth is at those positions on its orbit which give rise to midsummer and midwinter to an observer in the northern hemisphere.

At the midsummer position, E, the Sun is raised above the plane of the equator by an amount equal to the tilt of the Earth's axis,

which is 23°27′. At the midwinter position, E', the Sun is depressed an equal amount below the plane of the equator. The plane of the ecliptic is therefore inclined at an angle of 23°27′ to the plane of the equator, as shown in figure 53.

The First Point of Aries. The ecliptic cuts the celestial equator in two points. The one through which the Sun passes about the 21st March is called the *First Point of Aries* or the vernal equinox, and denoted by ♈, the ram's horns in the signs of the Zodiac; and the other through which the Sun passes about the 23rd September is called the *First Point of Libra,* or the autumnal equinox, and denoted by ♎, the scales.

The First Point of Aries takes its name from the constellation Aries through which the Sun appeared to pass when the early astronomers decided its path. There is, however, a slow backward

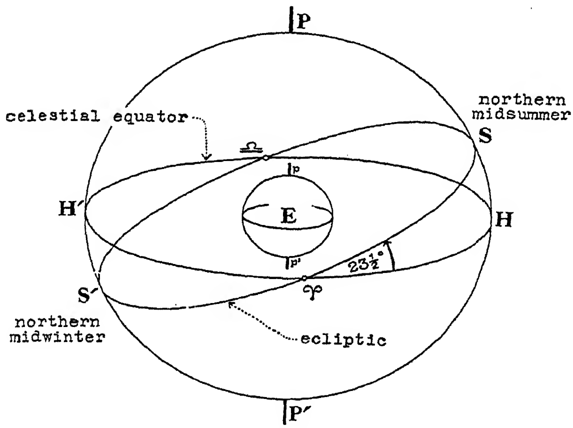


FIGURE 53.

movement of the actual point of intersection of the ecliptic and celestial equator along the ecliptic. The First Point of Aries therefore no longer coincides with the position of the constellation Aries in the celestial sphere.

The Celestial Meridians. These are semi-great circles joining the celestial poles, and they correspond exactly to the terrestrial meridians.

Position of Heavenly Bodies in the Celestial Sphere. For ordinary purposes the position of a heavenly body is fixed in the celestial sphere in relation to the celestial equator and a particular celestial meridian in the same way that the position of a place on the Earth is fixed in relation to the terrestrial equator and the particular meridian through Greenwich. The celestial meridian selected is the one through the First Point of Aries.

Right Ascension. The *right ascension* of a heavenly body is the angle between the meridian through the First Point of Aries and the meridian of the heavenly body, measured eastward from the former. It is thus the angle at the pole, or the angular distance along the equator, ♈*R* in figure 54, and it is conveniently expressed in units of time (hours, minutes and seconds), 24 hours being equivalent to 360°.

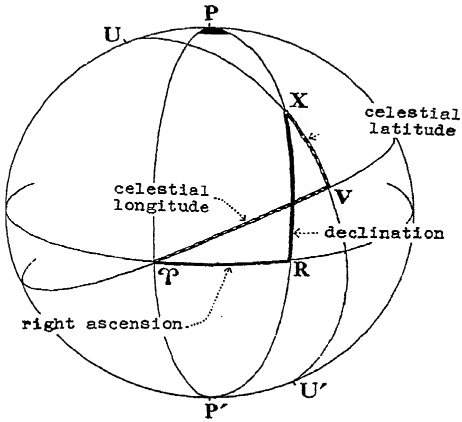


FIGURE 54.

Declination. This corresponds to terrestrial latitude and is the angular distance of the heavenly body north or south of the celestial equator—*RX* in figure 54—measured in degrees, minutes and seconds of arc.

Warning. Although right ascension and declination bear a strong resemblance to terrestrial longitude and terrestrial latitude, they should not be looked upon as celestial counterparts. Celestial longitude and celestial latitude are quantities quite different from right ascension and declination. Celestial longitude is measured along the ecliptic from the First Point of Aries to the meridian through the pole of the ecliptic (♈*V* in figure 54), and celestial latitude is measured from the ecliptic along this meridian (*VX* in figure 54). They do not occur in the theory of navigation.

The *Nautical Almanac*, 'abridged for the use of seamen', gives the right ascension and declination of all heavenly bodies of use to the navigator.

Because the Earth is turning continuously within the celestial sphere, the connexion between right ascension and longitude, and between declination and latitude, is never more than instantaneous.

Only once during the period of the Earth's rotation will the Greenwich meridian lie in the same plane as the meridian through the First Point of Aries and also directly beneath it.

Parallel of Declination. This corresponds to a parallel of latitude and is a small circle on the celestial sphere, the plane of which is parallel to the plane of the celestial equator.

Polar Distance. This is the angular distance of a body from the elevated pole; the pole, that is, above the observer's horizon. It is PX in figure 55, if the observer is assumed to be in north latitude.

When the elevated pole and the declination have the same names, the polar distance is clearly (90°—declination). When they have opposite names, it is equal to (90°+declination).

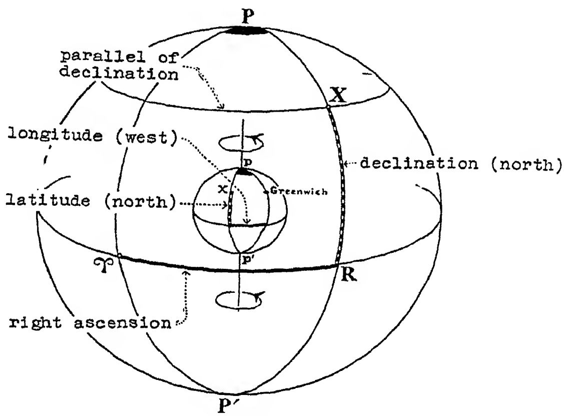


FIGURE 55.

Geographical Position. If a line is drawn from a heavenly body to the Earth's centre, the point where this line cuts the Earth's surface is called the geographical position of the body.

CX in figure 56 is such a line, and x is the geographical position of X. To an observer at x, the body would thus appear to be exactly overhead. That is, the body would be at the observer's *zenith*.

The Observer's Zenith. This, as indicated, is the point where the line joining the Earth's centre to the observer's position cuts the celestial sphere. The declination of the zenith (ZQ in figure 57) must therefore be equal the observer's latitude (Oq).

The Celestial or Rational Horizon. The great circle on the celestial sphere, every point of which is 90° from the observer's zenith, is known as the *celestial* or *rational horizon*. A plane through the centre of the Earth at right angles to the observer's radius, CO,

would cut the celestial sphere in this great circle. The celestial horizon therefore divides the celestial sphere into hemispheres, the upper one of which, containing Z, is known as the visible hemisphere

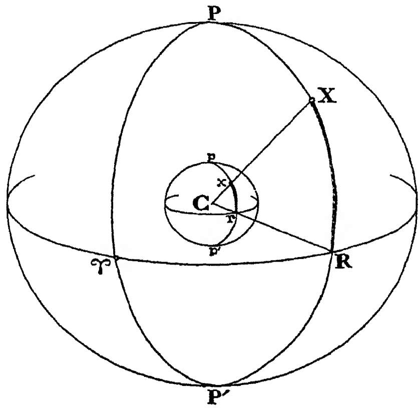


FIGURE 56.

because, subject to certain small adjustments described in the next chapter, all heavenly bodies in this half of the celestial sphere are visible to the observer at O. Heavenly bodies in the lower hemisphere cannot be seen by an observer thus situated.

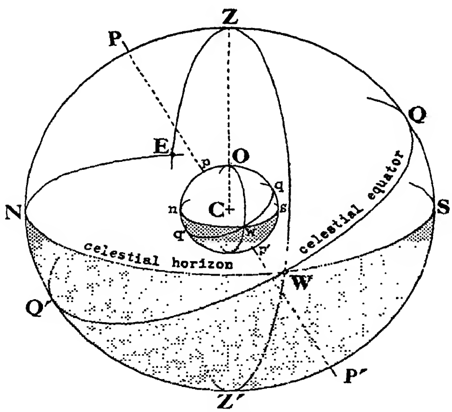


FIGURE 57.

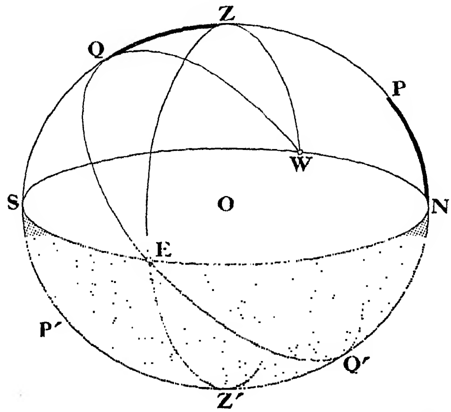
The Observer's Meridian. This is the celestial meridian which passes through the observer's zenith—$PZQSP'$ in figure 57. The meridian $PNQ'Z'P'$ differs from it by 12 hours in right ascension.

The points, N and S, in which these two meridians cut the

celestial horizon are the *north* and *south points,* the north point being the one nearer the north pole.

The *east* and *west points,* E and W, lie on the celestial horizon midway between N and S.

The Plane of the Meridian. Figure 57 is drawn on what is called the plane of the observer's meridian, or simply, the *plane of the*



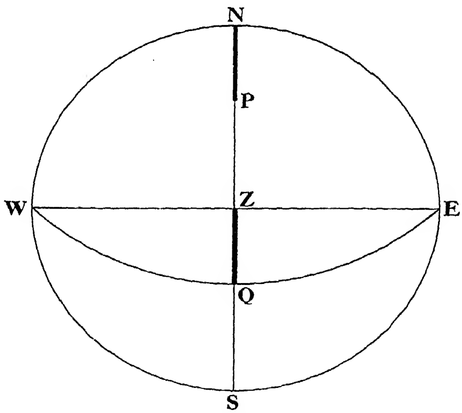
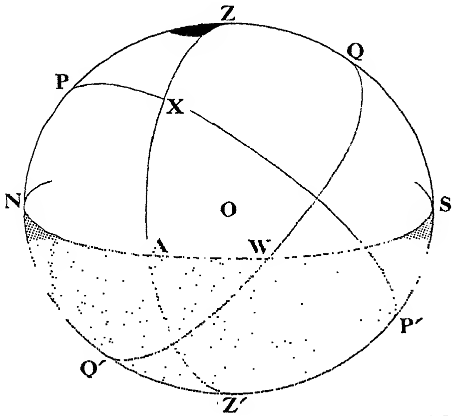


FIGURE 58a. FIGURE 58b.

meridian, the observer being in north latitude, and it shows the Earth and the celestial sphere as they would appear if seen from the west. From the east they would appear as shown in figure 58a, in which, for convenience, the Earth and the observer are indicated by the single point O.

The Plane of the Celestial Horizon. When it is convenient to show the whole visible sky, the figure must be drawn on the plane

of the celestial horizon as if the celestial sphere were seen from a position directly above the observer's zenith. Z therefore appears as the centre of the circle that is the celestial horizon; the north–south and east–west lines divide this circle into four equal parts, and the equator appears as a curve through W, Q and E as shown in figure 58b.



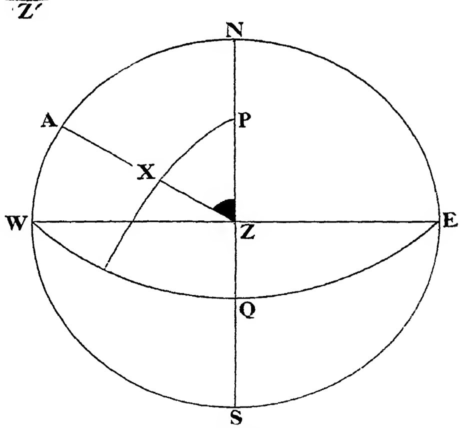


FIGURE 59a. FIGURE 59b.

The positions of P and Q are decided by simple proportion, and no attempt is made to insert them according to perspective.

In figure 58b, ZQ is the observer's latitude. But:

$$PZ+PN=90°=PZ+ZQ$$

i.e.

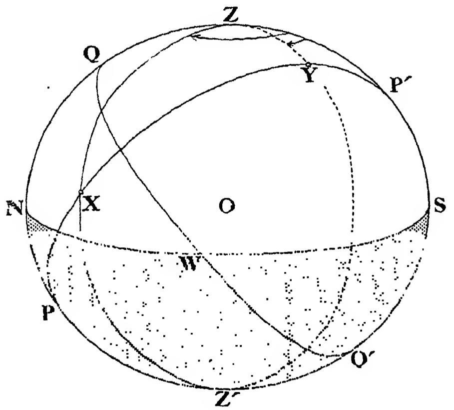
$$PN=ZQ$$

PN is therefore equal to the latitude, and the position of P is thus determined. If, for example, the observer's latitude is 35°N.,

P would be chosen so that, of the 90 equal parts into which NZ is divided, PN takes 35 and PZ the remainder.

Vertical Circles. All great circles passing through the observer's zenith are necessarily perpendicular to the celestial horizon and are known as vertical circles.

The Prime Vertical. The particular vertical circle passing through the east and west points is called the *prime vertical*.



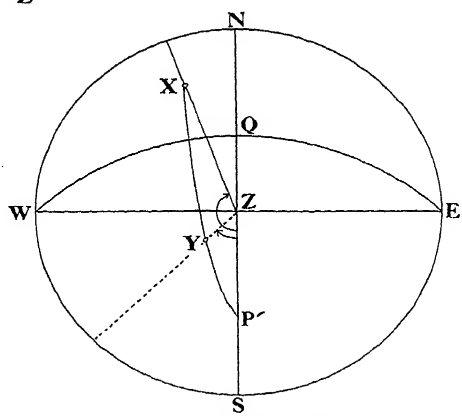


FIGURE 60a. FIGURE 60b.

The Principal Vertical Circle. The observer's meridian is sometimes called the *principal vertical circle* because it provides a fixed direction in the celestial sphere just as the observer's terrestrial meridian provides one on the Earth's surface.

The Azimuth of a Heavenly Body. This is the angle at the zenith between the observer's meridian and the vertical circle through the heavenly body, and it is measured east or west from his

meridian from 0° to 180°, and named ' N.' or ' S.' from the elevated pole.

Figures 59a and 59b show this angle, *PZX*, and should make it clear that the azimuth can also be measured from the points *N* or *S* along the celestial horizon, just as the d'long can be expressed either as an angle at the pole or as an angular distance along the equator.

Since the azimuth of a heavenly body is measured east or west from the meridian and named from the elevated pole, it is not always the same as the true bearing of the heavenly body, which is measured clockwise from true north. The azimuth of *X* in figures 59a and 59b is N.60°W., but the true bearing of *X* is 300°.

In figures 60a and 60b, which are drawn for an observer in south latitude, the azimuths of *X* and *Y* are respectively S.160°W. and S.45°W., whereas their true bearings are 340° and 225°.

The azimuth cannot be greater than 180°.

It should be noted also that when a figure is drawn on the plane of the horizon, only the azimuth is shown correctly to scale, Everything else, though conveniently put in by simple proportion, is wrongly placed. For practical purposes, however, the figure suffices because accurate measurements are not required from it.

CHAPTER VIII

ALTITUDE AND ZENITH DISTANCE

Although the position of a heavenly body is fixed in the celestial sphere by its declination and right ascension, these angular distances are measured from theoretical axes, and for his own convenience when he wishes to point out a particular heavenly body, an observer will employ others, his own meridian and the horizon, and will decide the heavenly body's position by a bearing from the meridian and an altitude above the horizon. But he will also make use of that altitude when finding his own position. He is therefore concerned not only with the altitude of the body above the horizon which he actually sees, but with its altitude above the celestial horizon; and having measured the altitude above the one, he applies certain corrections until it refers to the other.

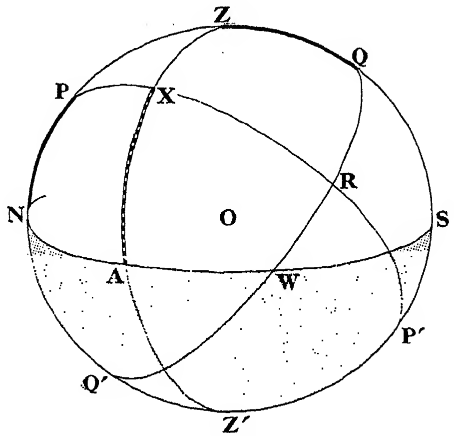


FIGURE 61a.

True Altitude of a Heavenly Body. This is the angular distance of the body above the celestial horizon, measured along the vertical circle through the body and the observer's zenith. In figure 61a (on the plane of the meridian) and 61b (on the plane of the horizon) the altitude of X is AX.

True Altitude of the Pole. It was shown on page 73 that PN is equal to ZQ, the observer's latitude. The *true altitude of the pole* (which is PN) is therefore the *latitude of the place* from which the observation is made.

The Observer's Sea or Visible Horizon. This is the horizon—usually the small circle on the Earth's surface where the sea and sky appear to meet—above which the observer actually measures the altitude of a heavenly body.

In figure 62, the tangent from the observer to the Earth's surface decides the position of this small circle, but refraction alters it slightly because the path of a light-ray from the horizon to the observer is not a straight line. (See page 81.)

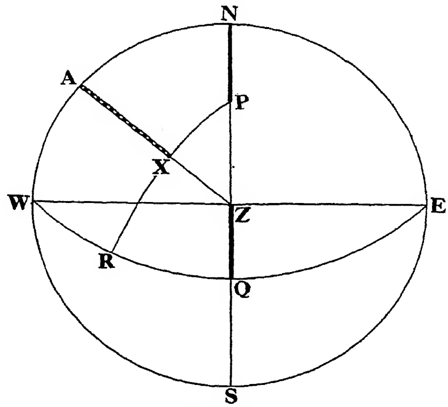


FIGURE 61b.

The Observed Altitude. An altitude above the sea-horizon (the angle DOX' in figure 62) is always known as an *observed altitude* in

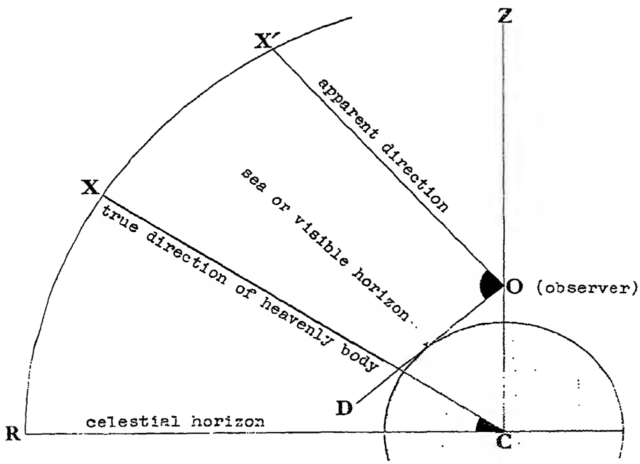


FIGURE 62.

order that it may not be confused with the *true altitude* (the angle RCX) which is obtained by correcting the observed altitude.

The Sextant Altitude. In practice, altitudes are measured with a sextant, and since most sextants give slightly erroneous readings, the

sextant altitude of a heavenly body will usually differ slightly from the observed altitude. This introduces another small correction.

Corrections to the Sextant Altitude. In order to obtain the true altitude, five corrections must be applied to the sextant altitude :

(1) index error.
(2) dip.
(3) refraction.
(4) semi-diameter.
(5) parallax.

For reasons which will be apparent, they are applied in that order.

Index Error. This is an instrumental error inherent in the sextant that is used for measuring the angle.

By a suitable arrangement of mirrors, described in Volume I, the sextant enables an observer to look directly at the horizon and,

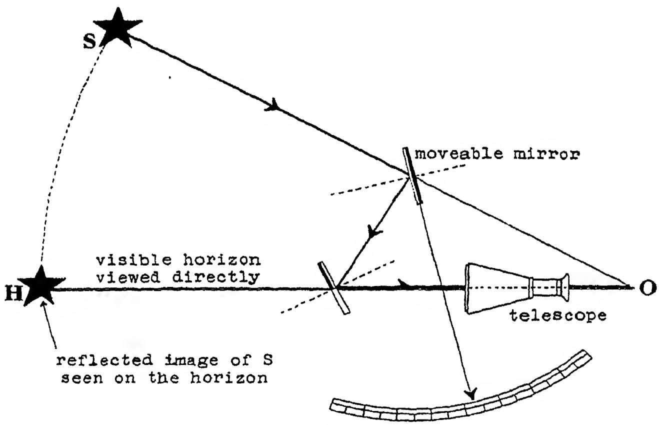


FIGURE 63.

by moving the arm of the sextant to which one of the mirrors is attached, to reflect the image of the body so that it appears to rest on the horizon. The position of the arm on the sextant arc indicates the altitude of the body—the angle *SOH* in figure 63.

When a heavenly (and therefore distant) body is viewed directly, the angle between it and its image as reflected in a sextant is nought, and the reading on the arc of the sextant should be nought. This, however, seldom occurs in practice because the mirrors are seldom in exact adjustment. The zero of the sextant scale, therefore, is not usually the true zero of the instrument. For example, if the sextant reading of the angle between the direct and reflected rays from the same distant object is 3′·0 on the arc, the instrument is reading 3′·0 too high and 3′·0 must be subtracted from all angles measured with the instrument. If, instead of reading 3′·0 too high, another sextant reads 2′·0 too low, 2′·0 must be added. These errors, denoted by

$-3'{\cdot}0$ and $+2'{\cdot}0$ respectively, are known as the *index errors* of the sextants. The methods of finding them are explained in Volume I.

Dip. *Dip* is the angle between the horizontal plane through the observer's eye and the apparent direction of the visible horizon. It is the angle DOH in figure 64.

Dip occurs because the observer's eye is always above sea-level and the observed altitude, DOX, is greater than the altitude would be to an observer at O' with no height of eye. The amount of dip therefore depends on the observer's height of eye, and it must be subtracted from the observed altitude.

In figure 64, it has been assumed that the heavenly body X is sufficiently distant for $O'X'$ to be considered parallel to OX. This assumption, however, does not take refraction into account.

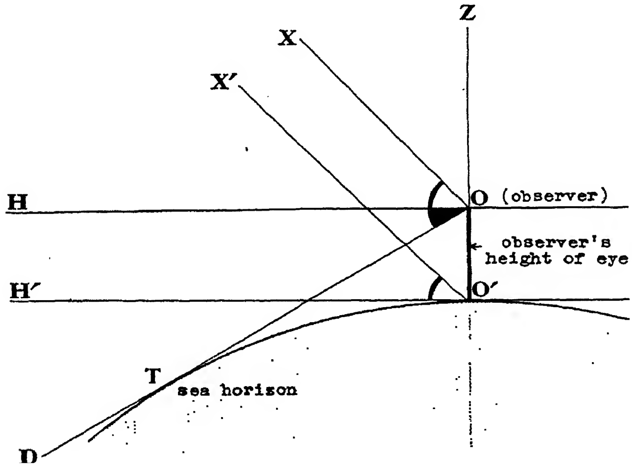


FIGURE 64.

Refraction. *Refraction* is the bending of light from its path that occurs when the ray passes from one medium to another of different density. The optical law—illustrated in figure 65—is that the ray is bent towards the normal on entering a more dense medium, the normal being the line perpendicular to the surface of the medium at the point where the ray enters.

A ray of light passing from an object at X and entering the water at N, for example, would be bent towards the normal LNM so that the angle ENM is less than the angle XNL. But an observer always sees an object in the direction in which the ray from that object enters his eye. To an observer at E, the object at X would therefore appear in the direction ENX', and the altitude of X appears to be increased.

Since the density of the air surrounding the Earth grows less as the distance from the Earth's surface increases, a ray of light from a heavenly body passes continually from one medium to another of

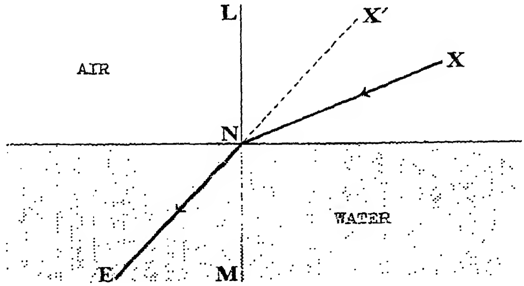


FIGURE 65.

greater density from the moment it enters the surrounding envelope of air until it reaches the observer. Its path is therefore curved as shown by the line $XTSRQO$ in figure 66.

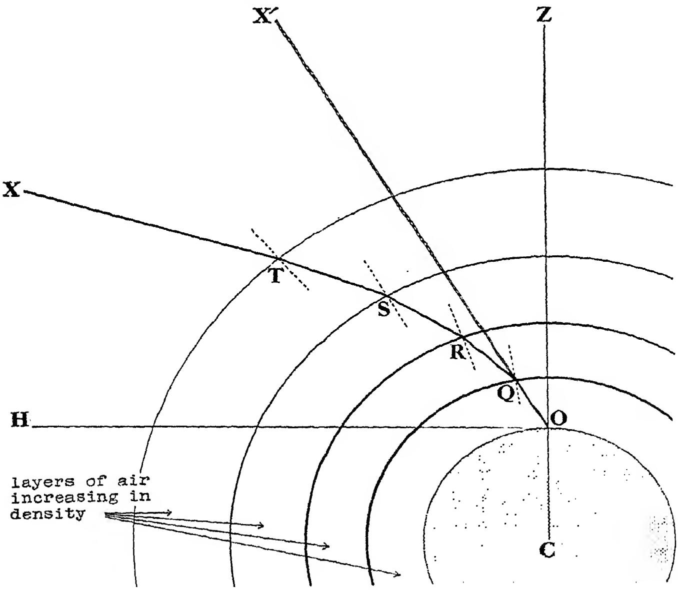


FIGURE 66.

For this reason an observer sees the heavenly body X as if it were at X', with an observed altitude HOX' which is greater than its actual altitude above the horizontal plane through O. The

difference between these two altitudes is the angle of refraction, and it must be subtracted from the observed altitude.

Refraction varies with the altitude. It is greatest when the body is on the horizon. It vanishes altogether when the body is at the observer's zenith, because the ray of light then passes along the normal itself.

Inman's Tables include tables of mean refraction, so called because the refraction is calculated for a barometric pressure of 29·94 inches or 1,015 millibars, and a temperature of 50° F. Further tables give the corrections to be applied to the mean refraction for other pressures and temperatures. In practice, however, these corrections are seldom applied because the low altitudes at which they become appreciable are avoided. As a general rule a navigator would not use an altitude less than 10° for finding his position.

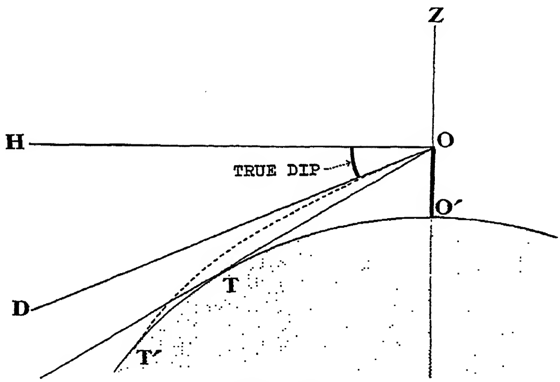


FIGURE 67.

Effect of Refraction on Dip. Since the effect of refraction is to increase the apparent altitude of the body, the horizon that an observer actually sees will not be the horizon formed by drawing tangents from his position at eye-level to the Earth's surface.

In figure 67, OT is the tangent from the observer to the Earth's surface, and T should mark the horizon. But the point which the observer actually sees is T', and, since the ray is bent as shown, the visible horizon appears in the direction OD, where OD is a tangent to $T'O$ at O. The dip is therefore the angle DOH, which is less than the theoretical angle of dip, TOH.

Inman's Tables give the values that the angle of dip, DOH, assumes for heights of eye up to 600 feet, so that in practice the effect of refraction on the position of the visible horizon (an effect known as terrestrial refraction) need not be considered.

Abnormal Refraction. The formula giving the dip is based on the assumption that there is not a great difference between the air

temperature and the sea temperature. If that difference is considerable, the tables are unreliable. There is no way of determining the extent of the error introduced, and if its existence is suspected, observations should be treated with caution.

Looming and mirage, phenomena described in Volume III, are signs of abnormal refraction.

Correction for Semi-Diameter. Because of their size and nearness to the Earth, an observer sees the Sun and Moon not as points of light but as bodies that have an appreciable diameter, and when finding their altitudes he measures the angles between the horizon and their lower or upper edges, and then arrives at the altitudes of their centres by adding or subtracting their semi-diameters. The positions of the Sun and Moon given in the *Nautical Almanac* are the positions of their centres.

The Sun's Semi-Diameter. The point on the Sun's disc nearest the horizon is called the Sun's *lower limb* and denoted by L.L. or $\underline{\odot}$,

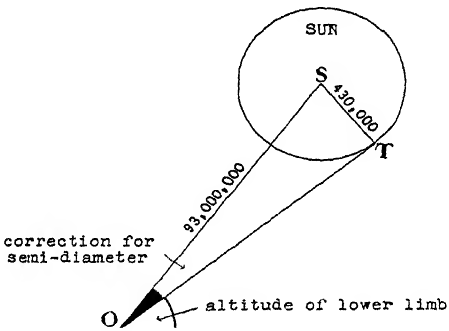


FIGURE 68.

and the point farthest from the horizon the *upper limb*, denoted by U.L. or $\bar{\odot}$. In practice, the altitude measured is generally that of the Sun's lower limb, and the Sun's semi-diameter is thus generally added.

The Sun's semi-diameter is measured by the angle TOS, in figure 68, where OT is a tangent and therefore at right-angles to the radius ST. ST itself is roughly 430,000 miles, and OS, the distance of the observer from the Sun's centre, which may be taken as the average distance of the Earth's centre from the Sun's, is 93,000,000. Hence :

$$\sin(TOS) = \frac{ST}{OS} = \frac{43}{9.300}$$

The average semi-diameter is thus 16′.

The *Nautical Almanac* gives the Sun's semi-diameter for each day. It varies between 15′45″ at the beginning of July when the

Earth is at its greatest distance from the Sun, to 16′18″ at the beginning of January when the Earth is at its least distance.

The Moon's Semi-Diameter. Unless the Moon is full, the observer usually has no choice but to measure the altitude of one limb, which may be either the upper or lower, because at any phase other than the full, only a part of the Moon is visible and the shape of that part is either gibbous or crescent with, more often than not, a sloping diameter.

Figure 69a shows a gibbous moon, the lower limb of which can be observed; figure 69b a crescent moon, the upper limb of which can be observed. In each figure ML is the semi-diameter, to be added to the altitude when the lower limb is observed and subtracted when the upper limb is observed.

The Moon's visible limb is always that which is nearer the Sun.

Augmentation of the Moon's Semi-Diameter. Since the Moon is relatively close to the Earth—the average distance between the

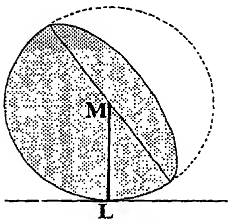


FIGURE 69a.

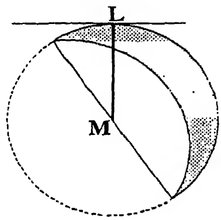


FIGURE 69b.

two centres is only 240,000′—the distance between an observer and the Moon varies appreciably with the Moon's altitude.

The Moon's semi-diameter to an observer at O, figure 70, is given by:

$$\sin (MOL') = \frac{ML'}{OM}$$

—where ML' is the Moon's radius and OM the distance of the observer from the Moon's centre, both lengths being measured in miles.

But to an observer situated at the Earth's centre, the Moon's semi-diameter would be given by:

$$\sin (MCL) \qquad ML$$

—where ML is equal to ML', and CM is the distance between the Earth's centre and the Moon's centre. This value of the semi-diameter is the one tabulated in the *Nautical Almanac* for each day.

Figure 70 shows that, so long as the Moon is above the horizontal tangent plane, OM is less than CM. The Moon's apparent

semi-diameter will therefore be greater than the one that is tabulated in the *Nautical Almanac*. The difference between them is known as *the augmentation of the Moon's semi-diameter*.

The amount of this augmentation is given in *Inman's Tables*. Its greatest value, when the Moon is overhead at M'', is $0'{\cdot}3$. It is therefore too small to be of any importance in the ordinary practice of navigation, and it is applied only when extreme accuracy is required.

Parallax. By applying the corrections so far considered to the observed altitude of a heavenly body's upper or lower limb, an

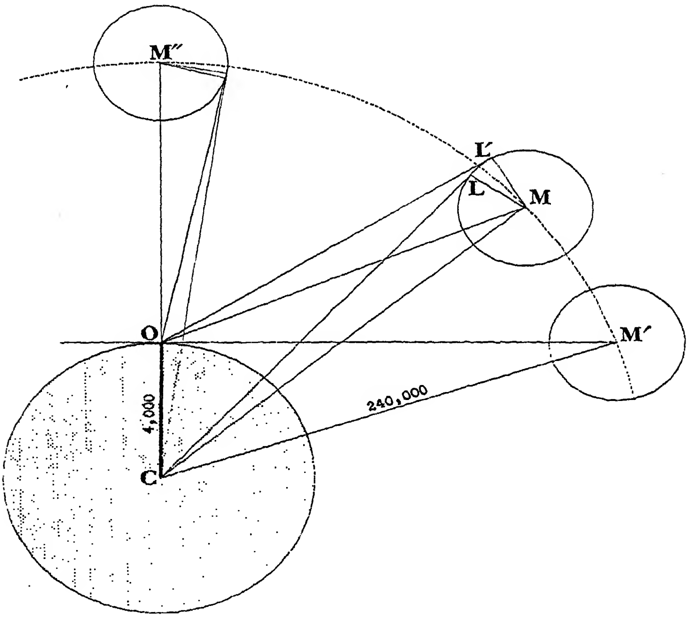


FIGURE 70.

observer is able to find the altitude of the heavenly body's centre above the horizontal tangent plane. It remains for him to allow for the Earth's radius.

In figure 71, the altitude above the horizontal tangent plane is HOX, and the true altitude is RCX.

Also, from the triangle OCX, since the external angle is equal to the sum of the two internal and opposite angles :

$$\angle VCX = \angle COX + \angle OXC$$

i.e. $$\angle RCX + 90° = \angle HOX + 90° + \angle OXC$$

$\therefore$ $$\angle RCX = \angle HOX + \angle OXC$$

The difference between the true altitude and the altitude above the horizontal plane is thus the angle OXC, which is the angle subtended at the heavenly body by the radius of the Earth drawn to the observer's position. This angle is known as the *parallax in altitude*, or simply the *parallax*, and it is clearly positive.

By the rule of sines applied to the triangle OCX :

$$\frac{\sin(\text{parallax})}{OC}=\frac{\sin(COX)}{CX}$$

i.e.

$$\sin(\text{parallax})=\frac{OC}{CX}\sin(90°+HOX)$$

$$=\frac{OC}{CX}\cos(\text{altitude})$$

The parallax of a heavenly body therefore vanishes when the body is at the observer's zenith.

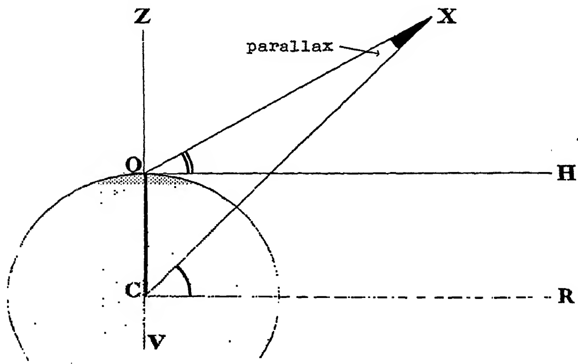


FIGURE 71.

Horizontal Parallax. Since the greatest value of the cosine is unity, and this value is attained when the angle is nought, the parallax of a heavenly body is greatest when the heavenly body is on the horizon, and it is then known as the *horizontal parallax*.

From figure 72 it is seen that the horizontal parallax is given by :

$$\sin(\text{H.P.})=\frac{OC}{CX},$$

Hence the formula giving the parallax becomes :

$$\sin(\text{parallax})=\sin(\text{H.P.})\cos(\text{altitude})$$

, as an approximation, since both the parallax and the horizontal parallax are small (see page 246 of the Appendix) :

$$\text{parallax}=(\text{H.P.})\cos(\text{altitude})$$

Only the Moon's horizontal parallax is large, and that is because the Moon is close to the Earth in comparison with other heavenly bodies, and the horizontal parallax is, from the above formula, governed entirely by the distance of the heavenly body from the

Earth. OC is constant, and therefore, as CX' increases, OC/CX' decreases.

For the Moon the horizontal parallax is about 60′ and the correction for parallax to be applied to the observed altitude is therefore appreciable.

For the planets the horizontal parallax is about 30″, for the Sun about 8″; and for the stars it is negligible.

Correction of the Sun's Altitude. Since the altitude of the Sun's lower limb is usually measured when a sight is taken at sea, the relevant corrections are grouped together and given as a *total correction* for different altitudes and heights of eye. In *Inman's Tables* the altitudes range from 5° to 90° and the heights of eye from $10\frac{1}{2}$ feet to $62\frac{3}{4}$ feet; and the semi-diameter used in compiling the total corrections is 16′. To allow for the small variation in the Sun's

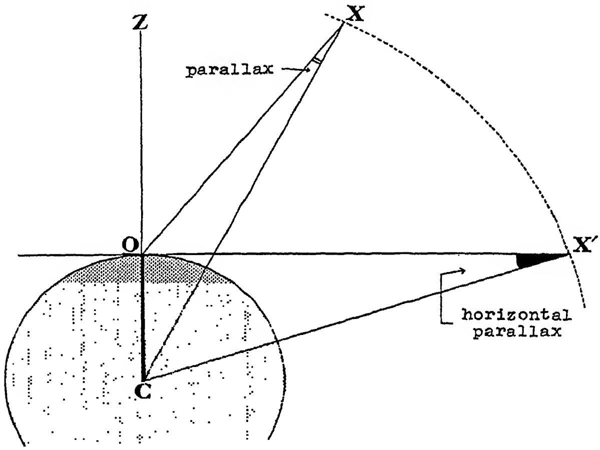


FIGURE 72.

semi-diameter during the year, a further correction is given at the bottom of the tables.

The sign of the total correction is given in the tables. It is, however, always plus for altitudes over the 10° laid down as the minimum for safe observation.

Suppose, for example, the sextant altitude of the Sun's lower limb is 36°20′·0 on the 1st September, and that the I.E. (index error) is —2′·7 and the observer's height of eye 32 feet. Then the true altitude would be found thus:

Sextant Altitude (Sext. Alt.) $\underline{\odot}$	36°20′·0
I.E.	—2′·7
Observed Altitude	36°17′·3
Sun's Correction (*Inman's Tables*)	+9′·1
True Altitude	36°26′·4

The separate components of this total correction are :

(1) dip	$-\ 5'{\cdot}6$	
(2) refraction	$-\ 1'{\cdot}3$	From *Inman's Tables* and
(3) semi-diameter	$+15'{\cdot}9$	the *Nautical Almanac*.
(4) parallax	$+\ 0'{\cdot}1$	

Observation of the Sun's Upper Limb. If the observation in the previous example should be taken of the Sun's upper limb, the total correction tables for the lower limb may still be used. It is necessary to subtract twice the Sun's semi-diameter from the corrected altitude, thus :

Sext. Alt. $\bar{\odot}$	36°20′·0
I.E.	−2′·7
	36°17′·3
Sun's Correction	+9′·1
	36°26′·4
Twice Semi-diameter	31′·8
True Altitude	35°54′·6

Correction of a Star's Altitude. Because it is so far from the Earth, a star has no parallax. For the same reason it appears as a point-source of light and has no semi-diameter. The total correction to its altitude therefore consists of dip and refraction only, and is negative.

Suppose the sextant altitude of Sirius is 17°49′·5, the I.E. being +3′·2 and the observer's height of eye 24 feet. Then the true altitude is found thus :

Sext. Alt. Sirius	17°49′·5
I.E.	+3′·2
	17°52′·7
Star's Correction	−7′·8
True Altitude	17°44′·9

The separate components of this total correction are :

(1) dip	−4′·8
(2) refraction	−3′·0

Correction of a Planet's Altitude. Unless an accuracy is required greater than that accepted in the practice of navigation, the planets are treated as stars and their altitudes corrected by means of the total correction table for stars.

Correction of the Moon's Altitude. The Moon's correction table, in *Inman's Tables,* is in two parts—one for observations of the upper

limb, and one for observations of the lower—and it allows for refraction, semi-diameter and parallax, but assumes a constant dip for a height of eye of 100 feet. For heights of eye other than 100 feet, corrections are given at the bottom of each page of the table.

It is possible to construct the table so as to allow for refraction, semi-diameter and parallax because the Moon's horizontal parallax is equal to a constant times the Moon's semi-diameter, that constant being the ratio of the Earth's radius to the Moon's radius. The arguments for the table are altitude and horizontal parallax, and the latter is given for each day in the *Nautical Almanac.*

Suppose the sextant altitude of the Moon's upper limb is 42°30′·0 on a day when the horizontal parallax is 58′·4 and the semi-diameter 15′·9, and that the I.E. is —2′·5 and the height of eye 32 feet. Then :

Sext. Alt. $\bar{☾}$	42°30′·0
I.E.	— 2′·5
	42°27′·5
Moon's Correction	+16′·5
Dip	+ 4′·2
True Altitude	42°48′·2

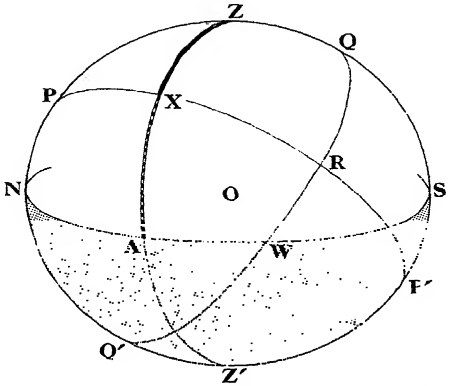
Had the corrections been applied separately, the working would have been :

Sext. Alt. $\bar{☾}$	42°30′·0
I.E.	— 2′·5
	42°27′·5
Dip and Refraction	— 6′·7
	42°20′·8
Semi-diameter	—15′·9
	42°04′·9
Parallax	+43′·3
True Altitude	42°48′·2

In order to find the parallax, the tables should be entered for an altitude of 42°05′. These show that the parallax is 43′ when the horizontal parallax is 58′, and 43′·8 when it is 59′. From the difference table for 0′·8, it is seen that the parallax corresponding to a horizontal parallax of 58′·4 is 43′·3.

Zenith Distance. Once the true altitude of a heavenly body has been obtained by the use of the appropriate corrections, the distance

of the body from the observer's zenith—ZX in figures 73a and 73b—can be found. It is $(ZA - XA)$ or ($90°$—altitude).



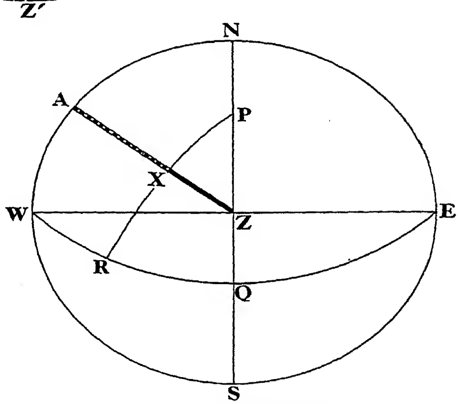


FIGURE 73a. FIGURE 73b.

This distance, ZX, is known as the *zenith distance,* and it is most important because it is the third side of the spherical triangle PZX.

CHAPTER IX

THE HOUR ANGLE

In studying the spherical triangle *PZX*, the three sides and the angle at *Z* have been considered. Of the remaining angles, *X*, the bearing of the observer from the heavenly body, is unimportant. The other, the angle at the pole, is known as the hour angle and is most important.

The Hour Angle of a Heavenly Body. This is defined as the angle between the observer's meridian and the meridian through the heavenly body. It is called the hour angle of the heavenly body

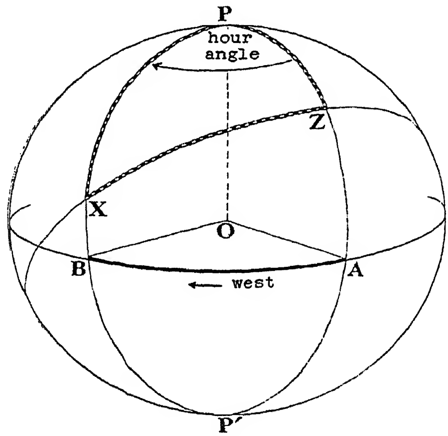


FIGURE 74.

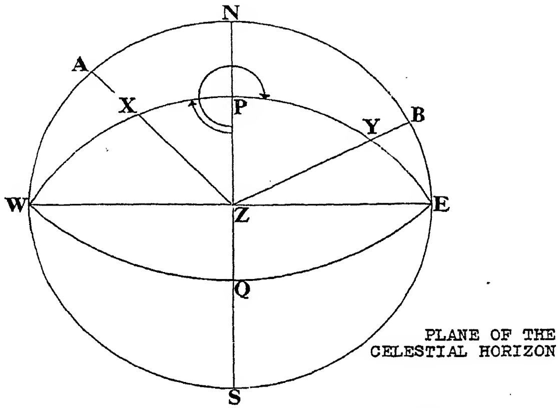
because it is an expression of what is understood as time, and it is measured westward from the meridian in units of time (0^h to 24^h) or units of arc (0° to 360°) according as convenience suggests.

Since the hour angle is an angle at the pole, the angular length of the arc *AB* on the celestial equator—figure 74—also measures it.

Effect of the Earth's Rotation. In Chapter VII it was explained how the Earth's steady rotation from west to east resulted in, to an observer on the Earth, an apparent and equally steady rotation of the celestial sphere from east to west. During one rotation of the

Earth, the hour angle of a heavenly body that is fixed in the celestial sphere will therefore increase from 0^h, when the heavenly body is on the observer's meridian, to 24^h, when it returns to this meridian. The hour angle of a heavenly body thus increases steadily throughout the day.

Relation between Azimuth and Hour Angle. When the hour angle of the heavenly body is less than 12^h, the heavenly body itself



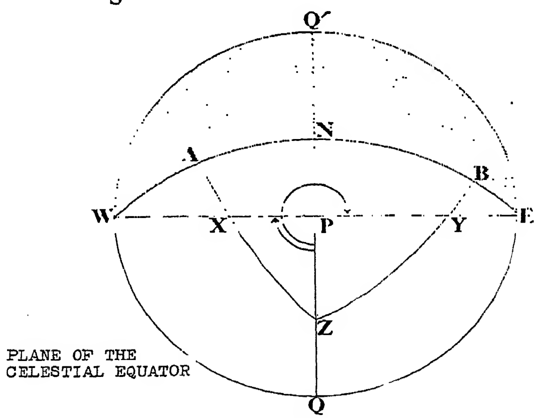


FIGURE 75a. FIGURE 75b.

must lie to the west of the observer's meridian ; when the hour angle is greater than 12^h, it must lie to the east. Hence, for hour angles between 0^h and 12^h the azimuth is west, and for hour angles between 12^h and 24^h the azimuth is east.

In figures 75a and 75b, which are drawn for an observer in north latitude, the azimuth of X is about N.45°W. and that of Y about

N.65°E., the hour angles of the two heavenly bodies being respectively 6^h and 18^h. Both X and Y therefore lie on the great circle passing through the pole and the east–west points. This particular great circle is sometimes known as the *six-hour circle*, and figure 75b is drawn on the plane of the celestial equator in order to emphasize it.

When the body is on the observer's meridian, it is neither east nor west, and its declination and the observer's latitude combine to determine whether its bearing is 0° or 180° ; that is, whether it lies north or south of the observer.

If the declination is greater than the latitude and of the same name, the heavenly body lies between the zenith and the elevated pole when the hour angle is 0^h. In these circumstances the bearing

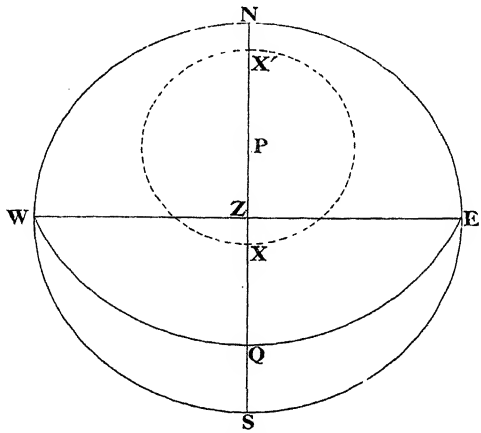


FIGURE 76.

of the heavenly body is north to an observer in north latitude, and south to one in south latitude.

Figure 76 shows that if the declination is less than the latitude, both names being north, the bearing of the heavenly body is south when the hour angle is 0^h. But the same heavenly body can also bear north, at X'. The hour angle is then 12^h, and the heavenly body is said to lie *below the pole*, a position discussed in detail on page 167.

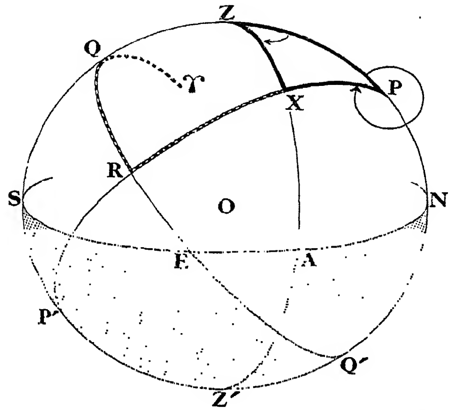
Position in the Celestial Sphere. It was stated at the beginning of the last chapter that, although the position of a heavenly body is fixed in the celestial sphere by its right ascension and declination, it can be decided by an altitude and azimuth. Figures 77a and 77b show the relation between the two methods.

ΥR is the right ascension and RX the declination ; AX is the altitude and PZX the azimuth.

The position of X would also be found by an hour angle (ZPX) and declination (RX), but this method is not used in practice.

In actual measurement, the hour angle of X is about 21^h, the azimuth about N.50°E., the altitude about 55°, the declination about 50°N., and the right ascension about 7^h.

Figure 77a, it should be noted, is drawn for an observer in north



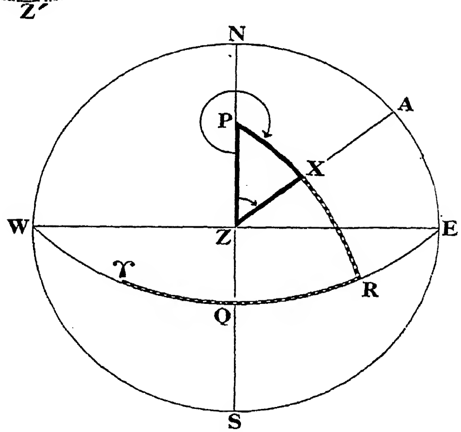


FIGURE 77a. FIGURE 77b.

latitude and with the east point showing. This is done to avoid having the heavenly body on the unseen hemisphere into which the observer's double meridian divides the Earth. The rules to be followed when a figure is to be drawn on the plane of the meridian therefore suggest themselves :

(1) *If the hour angle is less than* 12^h (that is, if the heavenly body is west of the meridian) the north point, N, should be put on the

left. A useful mnemonic is : Hour angle *less* than 12^h, north point on *left*.

(2) *If the hour angle lies between* 12^h *and* 24^h (that is, if the heavenly body is east of the meridian) N should be put on the right.

(3) *If the hour angle is* 0^h *or* 12^h (that is, if the heavenly body lies *on* the meridian or below the pole) N may be put either on the right or the left.

These rules still hold when the observer is in south latitude.

CHAPTER X

SOLAR TIME

The word *time* suggests not only duration but—as in the question : 'What is the time?'—a particular instant in that duration. Time may therefore be defined as the recurring division of duration. The lengths of the principal divisions, the year and the day, depend on certain astronomical phenomena, and the human race must accept them as they are. The lengths of the shorter divisions, the hour, minute and second, are chosen to suit man's convenience and are quite arbitrary subdivisions of the day.

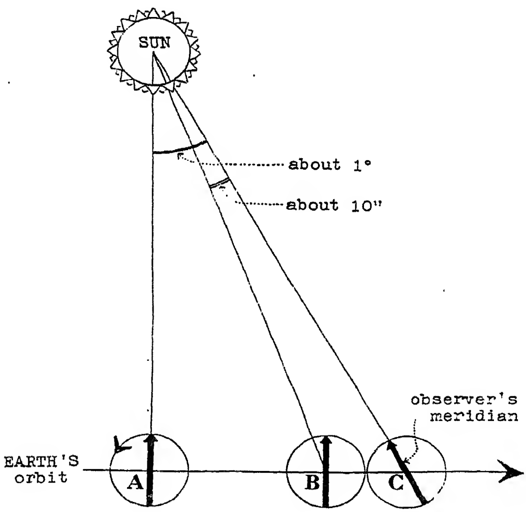


FIGURE 78.

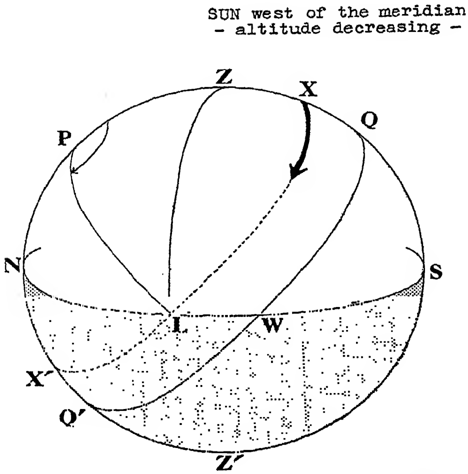
The uniform rotation of the Earth which results in the apparent and equally uniform rotation of the celestial sphere, so that heavenly bodies are continually crossing and returning to an observer's meridian, gives rise to the unit known as a day.

A day is the interval that elapses between two successive transits of some heavenly body across the same meridian.

All heavenly bodies are thus time-keepers, but, for reasons that will be seen, some are more convenient than others. The Sun is not a perfect time-keeper. The Sun, however, gives light and heat to

the Earth and so governs life on the Earth. The human race is therefore compelled to accept the Sun as the heavenly body by which the day is decided in ordinary human affairs.

The Apparent Solar Day. The interval that elapses between two successive passages or transits of the Sun across the same meridian is an *apparent solar day*.



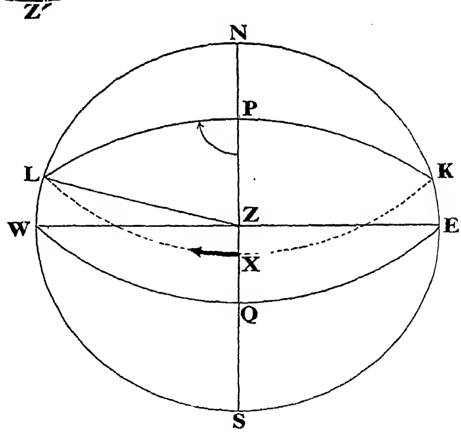


FIGURE 79a. FIGURE 79b.

The apparent solar day is not an interval of fixed length because the Earth does not move along its orbit round the Sun at a constant speed. Its speed is greatest when it is nearest the Sun, and least when it is farthest away. The distance it travels along its orbit in any fixed interval is therefore variable. The time taken for it

to make one complete revolution of 360° on its axis gives such a fixed interval, but that interval will not be the length of a day as defined by the Sun.

If the Sun is on the observer's meridian when the Earth is at A (figure 78) it will not be on that same meridian when the Earth has completed one revolution of 360° because the Earth will have moved along its orbit to B. Before the Sun is again on the observer's meridian, the Earth must turn still more on its axis, and it will have reached C when the transit occurs. The interval between two successive transits of the Sun across the observer's meridian is therefore the interval that elapses while the Earth is travelling along its orbit from A to C, a distance that is neither constant in length nor described in a constant time. [For the full explanation of this, see Chapter IX of Volume III.]

Apparent Noon. When the Sun is on the observer's meridian, the time is said to be *apparent noon* and, as figures 79a and 79b show, the Sun has then its greatest altitude, SX, above the observer's celestial horizon, the observer being assumed to be stationary.

The path along which the Sun appears to travel during the day, moving westward in the direction shown by the arrow, is the parallel of declination of which XX' is a part. The moment it leaves the meridian, it therefore approaches the horizon and its altitude decreases.

When the Sun reaches L, the point where the parallel of declination cuts the horizon, the altitude is 0° ; the zenith distance has increased to 90°, and the Sun is said to set.

NOTE. The obvious complications that arise because the observer, at what he calls sunset, watches the Sun's upper limb disappear below his visible horizon, whereas figures 79a and 79b show its centre on the celestial horizon, are dealt with fully in the chapter on the rising and setting of heavenly bodies.

Apparent Solar Time. The hour angle of the Sun at this moment is ZPL, the angle between the observer's meridian and the Sun's meridian, or the angle through which the Sun's meridian has appeared to move from its noon position, PZX.

Continuing westward along the parallel of declination, the Sun reaches K (figures 80a and 80b), at which point it is said to rise. Its hour angle then is ZPK.

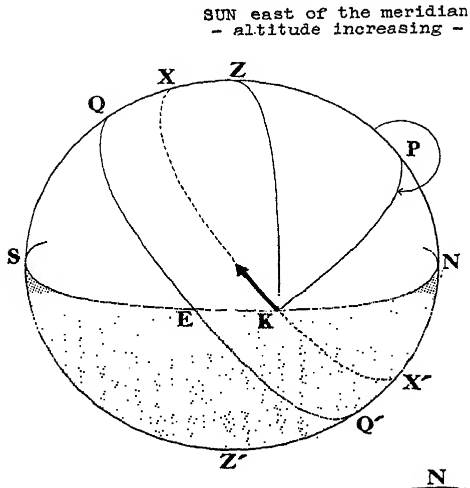
The value of the Sun's hour angle would thus seem to provide a convenient measure of apparent solar time, and it would do so but for the fact that the day would then begin and end at noon. For practical purposes a day beginning and ending at midnight is more suitable. Apparent solar time is therefore measured from midnight. Hence the definition :

Apparent solar time is the hour angle of the Sun $\pm 12^h$.

In figure 80b, the hour angle of the Sun at sunrise (ZPK) is about 17^h, but the apparent time of sunrise is about 5^h ; and in figure 79b

the hour angle of the Sun at sunset (ZPL) is about 7^h, but the apparent time of sunset is about 19^h.

The hour angle of the Sun is always referred to as the H.A.T.S. This is an abbreviation for Hour Angle of the True Sun, the Sun being so called in order that it may not be confused with the Mean Sun.



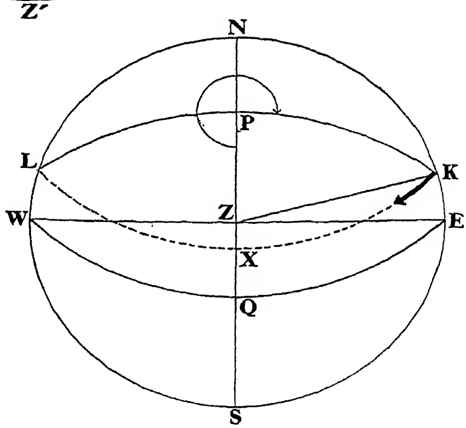


FIGURE 80a. FIGURE 80b.

The Mean Sun. It was stated at the beginning of this chapter that the apparent solar day is not a constant interval because the Earth's orbital motion is not constant. To an observer on the Earth this variable motion is revealed by corresponding variations in the apparent speed of the Sun along the ecliptic, and further variations are introduced when the Sun's motion is projected on to

the celestial equator where hour angles are measured. The hour angle of the True Sun does not, therefore, increase at a uniform rate, and it does not give a practical unit of measurement, which must be uniform.

To overcome this difficulty and yet maintain the connexion with the True Sun which is essential since the True Sun governs life on the Earth, a Mean Sun is introduced.

The Mean Sun is an imaginary body which is assumed to move in the celestial equator at a uniform speed round the Earth, and to complete one revolution in the time taken by the True Sun to complete one revolution in the ecliptic.

Mean Solar Time. This, by analogy with the definition of apparent solar time, is the *hour angle of the Mean Sun* $\pm 12^h$.

The hour angle of the Mean Sun is always denoted by H.A.M.S.

The Mean Solar Day. The interval that elapses between two successive transits of the Mean Sun across the same meridian is called a *mean solar day.*

In one mean solar day the Mean Sun moves westward from the meridian and completes one circuit of 360° in longitude in the 24 mean solar hours into which the period is divided. The rate of travel is thus 15° per mean solar hour.

The mean solar hour (called an hour for short) is further divided into 60 minutes, and these are in turn divided into 60 seconds. It must therefore be borne in mind that the units of time in everyday use are mean solar units.

The Civil Day. As the name suggests, the civil day is the day that suffices for human affairs. It begins at midnight when the Mean Sun makes its lower transit (that is when the H.A.M.S. is 12^h) and it ends at the next midnight. It is divided into 24 mean solar hours which are counted in two series of 12^h, the first marked A.M. (*ante meridiem*) and the second P.M. (*post meridiem*). The first therefore extends from midnight to noon, a period during which the H.A.M.S. lies between 12^h and 24^h, and the second from noon to midnight, when the H.A.M.S. lies between 0^h and 12^h.

The Astronomical Day. For tabulation purposes it is clearly more convenient to write 0915 and 2115, or 09^h15^m and 21^h15^m as in the *Nautical Alamnac,* instead of 9.15 a.m. and 9.15 p.m. A day reckoned in one series of hours from 0 to 24 is therefore introduced and referred to as the *astronomical day.*

Local Mean Time. (L.M.T.) This is the mean time kept at any place when the hour angle of the Mean Sun is measured from the meridian of that place. Hence the definition :

L.M.T. at any instant is the hour angle of the Mean Sun at that instant, measured westward from the meridian of the place, $\pm 12^h$.

Greenwich Mean Time. (G.M.T.) This is local mean time on the meridian of Greenwich, and it may be defined in analogous terms.

G.M.T. at any instant is the hour angle of the Mean Sun at that instant, measured westward from the Greenwich meridian, $\pm 12^h$.

In figure 81a, the L.M.T. at Greenwich is about 15^h, whereas the H.A.M.S. is 3^h. G.M.T. is thus (H.A.M.S.$+12^h$). In figure 81b, the L.M.T. at Greenwich is about $9\frac{1}{2}^h$, whereas the H.A.M.S. is $21\frac{1}{2}^h$. G.M.T. is thus (H.A.M.S.-12^h).

Longitude and Time. Since the positions of places on the Earth are fixed in relation to the Greenwich meridian, the longitude of a place must provide the connexion between L.M.T. at that place and L.M.T. at Greenwich. This fact is revealed at once in the time-difference which exists between Greenwich and, for example, New York.

The longitude of New York is roughly 75°W. The Mean Sun, travelling westward at 15° per hour, covers this angular distance in 5^h. New York is thus 5^h west of Greenwich.

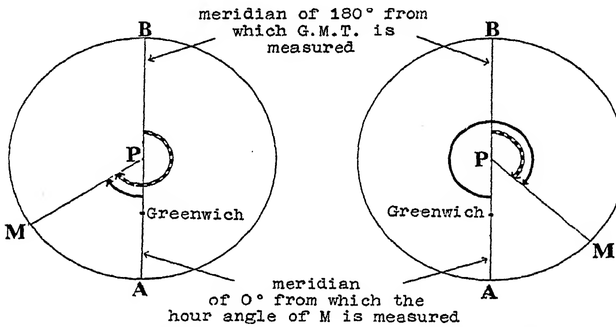


FIGURE 81a. FIGURE 81b.

When the Mean Sun reaches the New York meridian, its hour angle with reference to that meridian is 0^h. But to an observer at Greenwich, who measures the angle from the Greenwich meridian, its hour angle is 5^h because that is the period which has elapsed since the Mean Sun crossed the Greenwich meridian.

Similarly, when the Mean Sun is on the Greenwich meridian, its hour angle there is 0^h, but to an observer at New York its hour angle is 19^h—(24^h-5^h)—because 5^h must elapse before it reaches the New York meridian.

At any intermediate time when the Mean Sun is at *M*, figure 82, the Greenwich hour angle will be *APM*, which is about 3^h, and the L.M.T. at Greenwich will be 15^h; but the New York hour angle will be *BPM* (measured westward) which is about 22^h, and the L.M.T. at New York will be 10^h. The time of any event recorded by an observer at New York thus differs from the time recorded at Greenwich by the 5^h that form the time-equivalent of the d'long between the two places.

What has been said of New York in relation to Greenwich holds in principle if any other place is substituted for New York. To convert the L.M.T. on one meridian into the L.M.T. at Greenwich, it is therefore necessary to apply only the longitude expressed in time.

Rule for Converting L.M.T. to G.M.T. It is seen by comparing the times of the same event recorded by observers in New York and Greenwich, that L.M.T. at New York, a place 5^h *west* of Greenwich, is always 5^h behind or *slow* on L.M.T. at Greenwich. But it would be equally true to say that L.M.T. at Greenwich, a place 5^h *east* of New York, is 5^h ahead or *fast* on L.M.T. at New York. L.M.T. on any meridian is thus fast or slow on the L.M.T. on a second meridian according as the first meridian lies east or west of the second

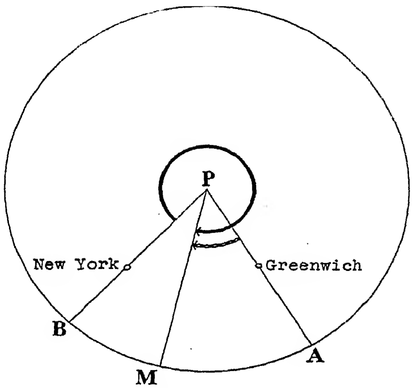


FIGURE 82.

meridian. When the second meridian passes through Greenwich, this relation between the two times is conveniently summarised thus :

Longitude west, Greenwich time best.
Longitude east, Greenwich time least.

If, for example, the G.M.T. of an observation is $19^h23^m43^s$ the L.M.T.s of the same observation at places in 48°W. and $22\frac{1}{2}$°E. are :

	At 48°W.		*At $22\frac{1}{2}$°E.*
	h m s		h m s
G.M.T.	19 23 43	G.M.T.	19 23 43
Long. W.	3 12 00	Long. E.	1 30 00
L.M.T.	16 11 43	L.M.T.	20 53 43

Standard Time. It is clearly impracticable for each place to keep the time of its own meridian. Nor is it practicable for places all

over the world to keep the same time. A compromise is therefore effected, and places in the same neighbourhood, which may be an entire country, elect to keep the same time. That time is usually based on a meridian running through the neighbourhood and differing in longitude from the Greenwich meridian by a convenient number of hours, and it is known as the standard time of the neighbourhood.

Zone Time. (**Z.T.**) This, in effect, is an extension of standard time to the sea, where it is impossible for a ship to keep the time of her meridian because, unless she happens to be steaming north or south, her meridian is always changing. The Earth is therefore divided into time-zones, each bounded by meridians 15° or 1^h in time apart, and situated so that the central meridian of each zone is an exact number of hours distant from the Greenwich meridian. The time kept in each zone is the time of its central meridian. Zone time therefore differs from G.M.T. by multiples of an hour, and is fast or slow according as the zone is east or west.

The zones themselves are numbered according to this difference on G.M.T.

Zone (0), for example, stretches from 7½°E. to 7½°W. The Greenwich meridian is central, and the time kept in the zone is therefore G.M.T. Zone (+1) stretches from 7½°W. to 22½°W., keeps the time of the meridian 15°W. and is 1^h slow on G.M.T., so that if this 1^h is added to the zone time the Greenwich time is obtained. Zone (—2) stretches from 22½°E. to 37½°E., keeps the time of the meridian 30°E. and is 2^h fast on G.M.T., so that if 2^h are subtracted from the zone time the Greenwich time is obtained.

These zones are given, together with certain geographical modifications, on a special Time Zone Chart which is incorporated and fully explained in Volume I.

Zone times are written thus :

Z.T. 1605(+2) 1st April
Z.T. 0510(—5) 9th July

—indicating that the approximate Greenwich times are 1805 and 0010 on these same dates.

The time that a ship keeps at sea in normal circumstances is zone time, and her clocks will be advanced or retarded as necessary to conform with the time of the various zones through which she passes. But they will not, as a rule, be altered the moment she enters a new zone. The fact that she crosses a dividing meridian and enters a new zone does not alter the time she is keeping. That alteration is not effected until her clocks are altered, and this will be done to suit the convenience of those on board. The fundamental purpose of zone time is the avoidance of confusion.

Two ships reporting the same submarine might give the time of sighting her as 1700 and 1535 respectively, and, unless the zone descriptions are attached, the person who receives these reports

would be led to suspect that there might be two submarines. If, however, one ship reports the event at 1700(—4) and the other at 1535(—3), it is seen from the Greenwich times, which are 1300 and 1235, that the reports most probably refer to the same submarine.

The important part of zone time is, therefore, not the actual zone in which the ship happens to be, but the zone description of the zone time which she is actually keeping. So long as that is given, confusion is avoided.

Greenwich Date. (G.D.) It is seen that any zone time is easily connected with G.M.T. by adding or subtracting the zone description. The G.M.T. thus obtained, when combined with the appropriate date, is called the G.D., or Greenwich Date. For example:

Z.T. 1605(+2) 1st April =G.D. 1805 1st April
Z.T. 0105(—7) 2nd April=G.D. 1805 1st April
Z.T. 0510(—5) 9th July =G.D. 0010 9th July
Z.T. 1610(+8) 8th July =G.D. 0010 9th July

British Summer Time. (B.S.T.) *British Summer Time* is a domestic adjustment whereby the civil population is given an hour's 'extra' daylight. In the summer months (for exact dates see the *Nautical Almanac*) clocks are advanced one hour so that during this period the navigator must treat all times as he would in Zone (—1). That is, he obtains a Greenwich Date by subtracting one hour.

Use of G.M.T. in Tabulation. In the *Nautical Almanac* and publications intended for use all over the world, it is clearly unnecessary, and impossible, to tabulate information relating to a given instant under sufficient headings (such as 1605(+2) 1st April, 0105(—7) 2nd April . . .) to enable any person to find it under the heading of the particular zone time which he happens to be keeping. Instead, that information is conveniently given under the appropriate G.M.T. heading (1805 1st April) because any zone time can be referred to the Greenwich meridian. G.M.T. is thus a 'standard' time for tabulation purposes.

Time-Keeping in a Ship. The ordinary clocks in a ship are adjusted as necessary for keeping zone time within a minute or two, and this is sufficiently accurate for domestic purposes and the life of the ship generally. But, since all information in the *Nautical Almanac* is given for G.M.T. throughout the year, it is necessary to know the accurate G.M.T. before the tables in the *Nautical Almanac* can be entered. Special clocks known as chronometers are therefore kept to give this accurate G.M.T.

Chronometers are never altered or disturbed, whatever zone time happens to be in use. They are checked daily by means of time signals, and their errors are tabulated as fast or slow on G.M.T. It is thus always possible to obtain the exact G.M.T. of an event, such as the observation of a heavenly body. (For details of the

method of checking these errors, see Chapter XI, Volume I, and for a description of the chronometer itself, see Chapter XIX, Volume III.)

Deck Watches. In order to avoid moving the chronometer, the time of an observation is actually taken with an ordinary but reliable watch called a *deck watch*. The older pattern keeps approximate G.M.T., and its exact difference from G.M.T. (known as its *error*) is found by means of wireless time-signals or comparison with the chronometer.

The *second-setting deck watch* is constructed so as to keep exact G.M.T. In this watch the 'second' hand is the outer hand, and the graduated rim over which it moves can be turned until any particular graduation is brought under it. The hand itself is not touched.

The advantage of the second-setting watch lies in its elimination of the error on G.M.T. and the consequent saving of a step in finding the hour angle. The watch, however, is still in the experimental stage, and for this reason the examples in this volume have been worked for watches of the older pattern.

Since deck watches (and chronometers) record a series of 12^h, they cannot distinguish between G.M.T. 3^h, say, and G.M.T. 15^h, and it is not always obvious from the 'time of day' in a ship whether the time is forenoon or afternoon at Greenwich. It may be midnight on the ship's meridian but midday at Greenwich. Should there be any uncertainty, it can be removed by comparing the G.M.T. obtained from the deck watch with the Greenwich date obtained from the zone time.

Suppose that, for example, at Z.T. 0530(—11) on the 20th September, the deck watch showed $6^h28^m43^s$ at the time of an observation, and that the deck watch was 31^s fast on G.M.T. Then :

Z.T.	0530	20th Sept. (approximate time from ship's clocks)
Zone	—11	
G.D.	1830	19th Sept. (approximate G.M.T.)

	h m s	
D.W.T.	6 28 43	
Error fast	31	
G.M.T.	18 28 12	19th Sept. (*not* $6^h28^m12^s$)

The G.M.T. must agree with the Greenwich Date, which is, in effect, merely a term denoting 'approximate G.M.T.'.

The Date Line. This is the boundary between Zone (—12) and Zone (+12). It does not follow the meridian of 180° exactly, but shapes itself to accommodate particular South Sea Islands in particular groups so that all the islands in a group keep one time. This, however, does not affect the adjustment of date that must be made when a ship crosses from one of these zones to the other.

That such an adjustment is necessary may be seen by considering

the hypothetical experience of a Schneider Cup machine going round the world in latitude 60° at 450 knots, the Sun's speed in longitude in that latitude. If the pilot starts at noon on a Monday, when the Sun is on his meridian, and flies west, the Sun will remain on his meridian. He will thus experience no day at all. Wherever he is it will be noon, and people will be thinking about their midday meal. But when he arrives back on his starting point, the people he left there will be thinking about Tuesday's meal. Somewhere during his journey, therefore, Monday has suddenly become Tuesday. That somewhere is the date line, on one side of which people are calling the time noon on Monday, and on the other noon on Tuesday.

A ship steaming round the world has a similar but less rapid and therefore not so obvious experience. She must adjust her clocks so as to keep the Sun approximately overhead at 1200, and when she crosses the date line she must make this further adjustment of one day.

Steaming west, a ship passes from a zone which is 12^h slow on G.M.T. to one which is 12^h fast the instant she crosses the date line. That is, instead of adding 12^h to the zone time, she must subtract 12^h. The effect of this, if no adjustment were made for date, would be to cause the Greenwich Date to go back 24^h, or one day. In order to avoid this discontinuity, it is necessary to miss one day in the local time and go from Tuesday to Thursday, say.

Steaming east, a ship goes from a zone which is 12^h fast on G.M.T. to one which is 12^h slow, and it is necessary to repeat a day in local time.

Example 1. *Ship Steaming West*

Consider a ship steaming from Vancouver to Sydney and arriving on the date line at 1345 on the 2nd June, a Tuesday. She passes from Zone (+12) to Zone (—12).

At the instant she leaves Zone (+12), her Greenwich Date is :

Z.T.	1345	Tuesday, 2nd June
Zone	+12	
G.D.	0145	Wednesday, 3rd June

If she makes no adjustment of one day, her Greenwich Date at the instant she enters Zone (—12) is :

Z.T.	1345	Tuesday, 2nd June
Zone	—12	
G.D.	0145	Tuesday, 2nd June

She is therefore a day out on Greenwich time, and in order to bring herself into step again she must call her zone time 1345(—12) Wednesday, 3rd June, so that :

Z.T.	1345	Wednesday, 3rd June
Zone	—12	
G.D.	0145	Wednesday, 3rd June

That is, she drops a day, and this she would probably do by keeping the time of Zone (+12) until midnight on Tuesday and then calling the next day Thursday in Zone (−12).

Example 2. *Ship Steaming East*

Suppose the ship were steaming from Sydney to Vancouver and arrived on the date line at the same time and date. She passes from Zone (−12) to Zone (+12).

At the instant she leaves Zone (−12), her G.D. is :

Z.T.	1345	Tuesday, 2nd June
Zone	−12	
G.D.	0145	Tuesday, 2nd June

If she makes no adjustment of one day, her G.D. at the instant she enters Zone (+12) is :

Z.T.	1345	Tuesday, 2nd June
Zone	+12	
G.D.	0145	Wednesday, 3rd June

She is therefore a day out on Greenwich time, and in order to bring herself into step again she must call her zone time **1345(+12)**, Monday, 1st June, so that :

Z.T.	1345	Monday, 1st June
Zone	+12	
G.D.	0145	Tuesday, 2nd June

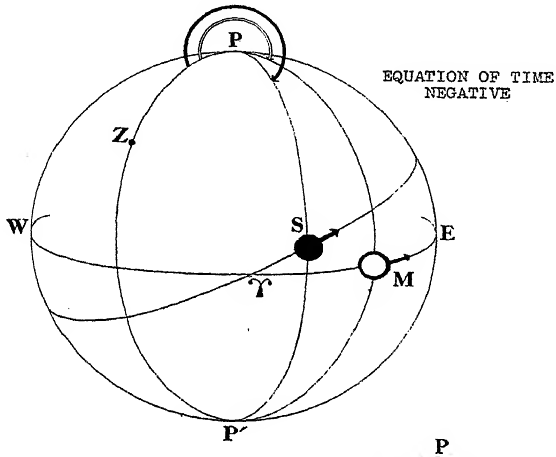
That is, she must repeat a day, and this she would probably do by keeping the time of Zone (−12) until midnight on Tuesday, 2nd June, and then repeating that day.

The Equation of Time. Although the assumption of an imaginary Mean Sun makes the ordinary clock possible, this same assumption also gives rise to a problem. The navigator seeking to fix his position by observing the Sun necessarily measures the altitude of the True Sun, and the True Sun keeps true or apparent solar time. But he notes the instant of this observation from a watch that keeps mean solar time. He must therefore be able to connect mean solar time with apparent solar time. This connexion is known as the *equation of time*.

The equation of time is defined as the excess of mean time over apparent time ; that is, H.A.M.S.—H.A.T.S.

The steps by which the equation of time is found are explained in Volume III. For the present it is sufficient to realise that, since the True Sun moves with a varying speed in the ecliptic and the Mean Sun moves with a constant speed in the celestial equator, they will not keep in step. At certain times of the year H.A.T.S.

will be greater than H.A.M.S., as shown in figure 83a. At other times it will be less, as shown in figure 83b. The equation of time may therefore be either positive or negative. About the 15th April, 14th June, 1st September and the 24th December it vanishes and changes sign.



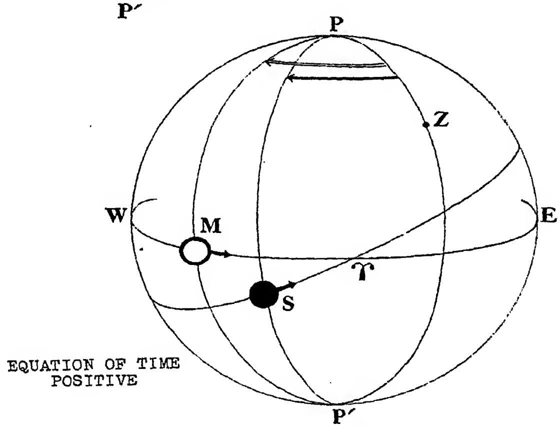


FIGURE 83a. FIGURE 83b.

The Quantity E. In the abridged *Nautical Almanac* the equation of time is not given directly, but as a quantity E which is defined by :

$$E = 12^h - \text{equation of time}$$

Since the equation of time never exceeds 17^m, E is always positive.

H.A.T.S.=L.M.T.+E. This equality, giving the hour angle of the True Sun, follows at once from the definition of L.M.T., which is:

$$\text{L.M.T.} \pm 12^h = \text{H.A.M.S.}$$

This may be written:

$$\text{L.M.T.} + 12^h - 24^h = \text{H.A.M.S.}$$

—or simply:

$$\text{L.M.T.} + 12^h = \text{H.A.M.S.}$$

—because no date can be attached to an hour angle, and this subtraction of 24^h (if necessary) does not affect the actual distance of the Mean Sun from the observer's meridian, any more than the subtraction of 360° affects the actual course of a ship. But H.A.M.S. and H.A.T.S. are connected by the relation:

$$\text{H.A.M.S.} - \text{equation of time} = \text{H.A.T.S.}$$

Therefore, to obtain H.A.T.S. from L.M.T. it is necessary:

(1) to add 12^h.
(2) to subtract the equation of time.

The quantity E, which is (12^h—equation of time), enables these two steps to be performed simultaneously. Thus:

	h m s		h m s
L.M.T.	16 12 23	L.M.T.	16 12 23
	+12	E	+11 50 00
H.A.M.S.	4 12 23	H.A.T.S.	4 02 23
E. of T.	−10 00		
H.A.T.S.	4 02 23		

Similarly, if L.M.T. is required from H.A.T.S. it can be found from:

$$\text{H.A.T.S.} - \text{E} = \text{L.M.T.}$$

To Find the H.A.T.S. from the D.W.T. It is now possible to find the hour angle of the True Sun from the time recorded by a deck watch at the moment of observation because the following connected facts are known:

(1) Zone time, measured from 0^h to 24^h, is the time kept by the ship's clocks, and is approximately a whole number of hours fast or slow on G.M.T. By the addition or subtraction of this number of hours from the zone time, the Greenwich Date or approximate G.M.T. is found. This is correct within a minute or so.

(2) From the deck watch with which the exact time of an observation is taken, the accurate G.M.T. of the observation is obtained by correcting the D.W.T. for the error on G.M.T. and noting from the Greenwich Date whether the time in question lies between 0^h and 12^h or between 12^h and 24^h.

(3) L.M.T. is the time, measured from 0^h to 24^h, of the particular meridian concerned in the problem, and it differs from G.M.T. by

the longitude of that meridian expressed in time, the longitude being applied according to the rule :

Longitude west, Greenwich time best.
Longitude east, Greenwich time least.

(4) L.M.T. is connected with the H.A.T.S. by a quantity E which is tabulated in the abridged *Nautical Almanac,* this quantity being added to the L.M.T. to give the H.A.T.S.

The procedure, in summary, is :

(1) Find the G.D.
(2) Find the G.M.T. and date.
(3) From the *Nautical Almanac* take out E for the G.M.T. and date.
(4) Apply the longitude of the place to the G.M.T. and so obtain the L.M.T.
(5) Add E to the L.M.T. and so obtain the H.A.T.S.

The following example shows how the work is set out in practice.

At Z.T. 1600(+11) *on the* 2nd *April* 1937 *the deck watch showed* $2^h59^m30^s$ *when an observation of the Sun was taken. The ship was assumed to be in longitude* 158°15′*W., and the deck watch was* 25^s *slow on G.M.T.*

Z.T.	1600 2nd April	
Zone	+11	
G.D.	0300 3rd April	

	E
	h m s
	11 56 27·8
	+0·7
	11 56 28·5

	h m s	
D.W.T.	2 59 30	
Error slow	25	
G.M.T.	2 59 55	3rd April
Long W.	10 33 00	
L.M.T.	16 26 55	
E	11 56 29	(to nearest second)
H.A.T.S.	4 23 24	

It will be noted that E is obtained by interpolating between the two elements taken from the *Nautical Almanac* for the G.M.T.s of 2^h and 4^h on the 3rd April. The difference between these elements is $1^s{\cdot}5$. The actual G.M.T. of the observation is almost exactly midway between 2^h and 4^h ; that is, half of $1^s{\cdot}5$ must be added to the value of E at 2^h.

The value of E is taken to the nearest second because there is no point in taking it to a degree of accuracy greater than that of the other quantities with which it is combined. In the abridged *Nautical Almanac* the quantities E and R (see Chapter XI) are given

to one decimal of a second in order to increase the accuracy of interpolations.

Connexion between Units of Time and Units of Arc. All important tables in *Inman's Tables* are constructed for angles expressed in units of time as well as units of arc, and it is possible to see at a glance how many hours, minutes and seconds are equivalent to a given number of degrees. If, however, these tables are not available, the conversion may be effected by remembering that, since the Sun completes its apparent revolution of 360° of longitude in 24^h :

$$15^\circ \equiv 1^h$$
$$1^\circ \equiv 4^m$$
$$15' \equiv 1^m$$
$$1' \equiv 4^s$$

These simple facts explain the following methods :

Arc into Time

Method I. Multiply by 4 *and divide by* 60.

If, for example, the angle is 117°34′, the work (which can be done in the head) would be :

$$4 \times 34' \div 60 = 136' \div 60 \equiv 2^m 16^s$$
$$4 \times 117^\circ \div 60 = 468^\circ \div 60 \equiv 7^h 48^m$$
$$\therefore \quad 117^\circ 34' \equiv 7^h 50^m 16^s$$

Method II. Divide by 15 *and multiply the remainder by* 4

$$117^\circ = (7 \times 15^\circ) \text{ and } 12^\circ \text{ over} \equiv 7^h 48^m$$
$$34' = (2 \times 15') \text{ and } 4' \text{ over} \equiv 2^h 16^s$$
$$\therefore \quad 117^\circ 34' \equiv 7^h 50^m 16^s$$

Time into Arc

Multiply the hours by 15 *and divide the minutes and seconds by* 4.

If, for example, the angle in time is $7^h 50^m 16^s$, the procedure would be :

$$7^h \equiv 7 \times 15^\circ = 105^\circ$$
$$50^m \equiv \frac{50^\circ}{4} = 12^\circ 30'$$
$$16^s \equiv \frac{16'}{4} = 4'$$
$$7^h 50^m 16^s \equiv 117^\circ 34'$$

CHAPTER XI

SIDEREAL TIME

In the last chapter it was stated that a day is the interval between two successive transits of some heavenly body across an observer's meridian, and that all heavenly bodies are therefore time-keepers.

The Sidereal Day. If the heavenly body selected is a star, the interval is known as a sidereal day to distinguish it from the solar day. For convenience, the First Point of Aries is taken instead of

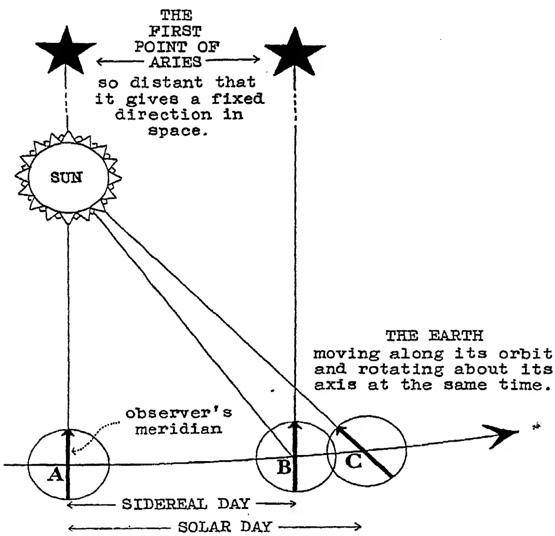


FIGURE 84.

an actual star, and the sidereal day is therefore defined as the interval between two successive transits of the First Point of Aries across the same meridian.

Figure 84 shows that the sidereal day is shorter than the solar day.

The First Point of Aries is on the meridian when the Earth has turned through 360° and reached *B*. The Sun is on the meridian when the Earth has turned through about 361° and reached *C*.

The sidereal day is thus shorter than a solar day by about 4 minutes, the time that the Earth takes to turn this extra degree, and the difference between the arcs AC and AB affords a graphical representation of the difference between the two kinds of day, although the arcs themselves are not convenient measures of them because the Earth's orbital velocity varies.

The sidereal day, however, is not a practical unit in a world that is ruled by the Sun, and except in observatories where some method of obtaining a uniform interval is essential, it is unimportant. Sidereal time, on the other hand, is important, because it enters the problem of finding a star's hour angle.

Local Sidereal Time. This is defined as the hour angle of the First Point of Aries, measured westward from the meridian of any place.

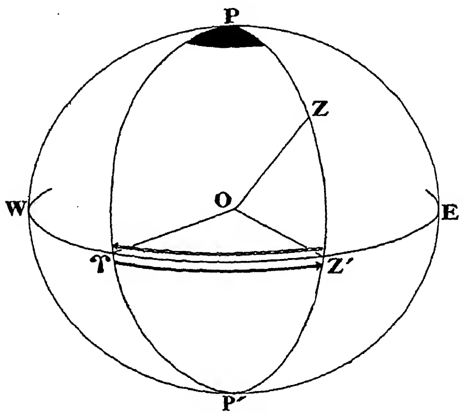


FIGURE 85.

Sidereal time is thus an expression of the angular distance of the First Point of Aries from an observer's meridian, and the observer is concerned with it only as that angular distance. He is not concerned with it as an expression of the amount of duration that has elapsed since the First Point of Aries crossed his meridian. If he were, he would at once become involved with the sidereal hour and sidereal minute, time units derived from the sidereal day and differing slightly from mean solar units ; and the connexion between the two sets of time-units would have to be considered. This labour, however, is avoided when he treats sidereal time as an angular distance because the units of arc in which he expresses it are constant units, suitable for expressing the angular distance of any heavenly body from his meridian.

In figure 85, $Z'\Upsilon$ measures the hour angle of the First Point of Aries, and this, numerically, is equal to $\Upsilon Z'$, which measures the right ascension of the meridian through Z. Sidereal time can there-

fore be regarded as the right ascension of the observer's meridian, a quantity referred to as the R.A.M. Thus :

$$\text{H.A.}\,\Upsilon = \text{sidereal time} = \text{R.A.M.}$$

In this fact lies the reason for choosing the First Point of Aries as the sidereal time-keeper. All stars are fixed in relation to this point by their right ascensions. If, therefore, the right ascension of the meridian is known, all stars can be fixed in relation to this

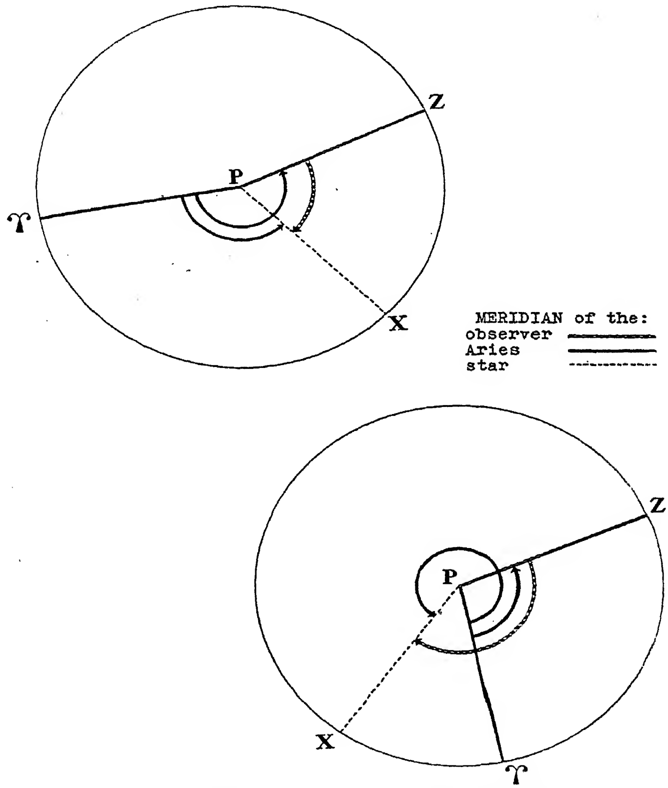


FIGURE 86a. FIGURE 86b.

meridian, the difference between the R.A.M. and the right ascension of any star being the hour angle of that star. (Figures 86a and 86b.)

In both these figures the right ascension of the star is ΥX, and the R.A.M. is ΥZ, measured eastward, or anti-clockwise. The hour angle of the star is ZX, measured westward, or clockwise.

Figure 86a shows that :

$$\text{R.A.M.} = \text{H.A.}\,\ast + \text{R.A.}\,\ast$$

Figure 86b shows that :

$$R.A.M. = H.A.\ast + R.A.\ast - 24^h$$

Hence, in general :

$$R.A.M. = H.A.\ast + R.A.\ast - 24^h \text{ (if necessary)}$$

That is, since the addition of 24^h or $360°$ does not affect the actual angular distance which is the hour angle :

$$H.A.\ast = R.A.M. - R.A.\ast$$

The Right Ascension of the Mean Sun. (R.A.M.S.) From the above relation it is seen that the R.A.M. must be found before the hour angle of the star can be found. The motion of the Mean Sun in the celestial equator enables this to be done.

If the Mean Sun is substituted for the True Sun in figures 50 and 51 on pages 66 and 67 of Chapter VII, it will be seen that this motion amounts to a steady increase in right ascension during the year, because the Mean Sun moves at a steady speed round the celestial equator along which right ascensions are measured as angular distances.

The point X, which also lies on the celestial equator in figures 86a and 86b, may therefore be taken as the position of the Mean Sun at the moment under consideration ; and when this is done it is at once apparent that similar relations hold for the hour angle of the Mean Sun and the R.A.M.S. as for the star's hour angle and right ascension.

Figure 86a then shows that :

$$R.A.M. = H.A.M.S. + R.A.M.S.$$

Figure 86b shows that :

$$R.A.M. = H.A.M.S. + R.A.M.S. - 24^h$$

Hence, in general :

$$R.A.M. = H.A.M.S. + R.A.M.S. - 24^h \text{ (if necessary)}$$

The exact hour angle of the Mean Sun is found from the deck-watch time of the observation. The *Nautical Almanac* gives the R.A.M.S. (indirectly as a quantity R) and therefore the exact position of the Mean Sun for any G.M.T.

The R.A.M. is thus expressed in terms of the Mean Sun's hour angle and right ascension, and also in terms of the star's hour angle and right ascension. By equating these two sets of terms, the star's hour angle is given as the sum and difference of known quantities.

$$H.A.\ast + R.A.\ast - 24^h \text{ (if necessary)}$$
$$= H.A.M.S. + R.A.M.S. - 24^h \text{ (if necessary)}$$

Or simply :

$$H.A.\ast + R.A.\ast = H.A.M.S. + R.A.M.S.$$

The hour angle of the star is therefore given by :

$$H.A.\ast = H.A.M.S. + R.A.M.S. - R.A.\ast$$

Figure 87 shows the relative positions of the meridians of the Mean Sun, the star, the observer and the First Point of Aries when this equation is satisfied without the introduction of 24^h.

Since the right ascension of the meridian is essentially an hour angle, it has no date, and the addition or subtraction of 24^h (or 360°) does not alter the actual angular distance of the First Point of Aries from the meridian. The terms in 24^h are therefore suppressed in the above equation, but it must be understood that where the addition of two quantities gives an angle greater than 24^h, 24^h must be subtracted, and where a larger angle is subtracted from a smaller, 24^h must be added to the smaller to keep the angle positive. Figures 88a and 88b show the relative positions of the various meridians when this addition and subtraction must be made.

The Quantity R. The procedure for finding the hour angle of a star is shown in the following example, and it is seen that a step is saved if, instead of the R.A.M.S., a quantity (R.A.M.S.$\pm 12^h$) is tabulated. This is the quantity R, and the sign is chosen so that R is always positive.

At Z.T. 2035(0) *on the* 3*rd April* 1937 *in longitude* 3°16′*W. the deck watch showed* $8^h34^m35^s$ *when an altitude of Achernar was observed. The deck watch was* 4^s *fast on G.M.T. What was the hour angle of Achernar?*

Z.T.	2035
Zone	0
G.D.	2035 3rd April 1937

	h m s
D.W.T.	8 34 35
Error fast	04
G.M.T.	20 34 31 3rd April
Long. W.	13 04
L.M.T.	20 21 27
	±12
H.A.M.S.	8 21 27
R.A.M.S.	0 47 00
R.A.M.	9 08 27
R.A.✳	1 35 21
H.A.✳	7 33 06

R
h m s
12 46 54·5
+5·7
12 47 00·2

	h m s
L.M.T.	20 21 27
R	12 47 00
R.A.M.	9 08 27
R.A.✳	1 35 21
H.A.✳	7 33 06

The value of R is taken to the nearest second for the same reason that the value of E is taken to the nearest second.

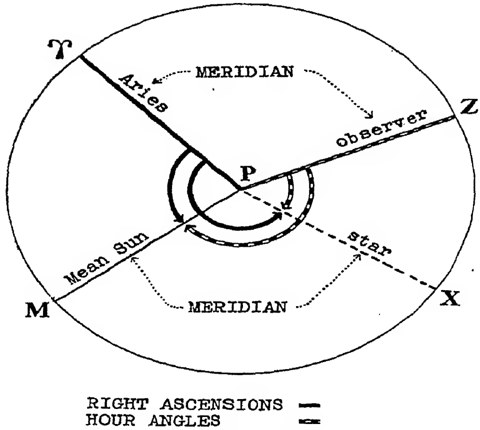


FIGURE 87.

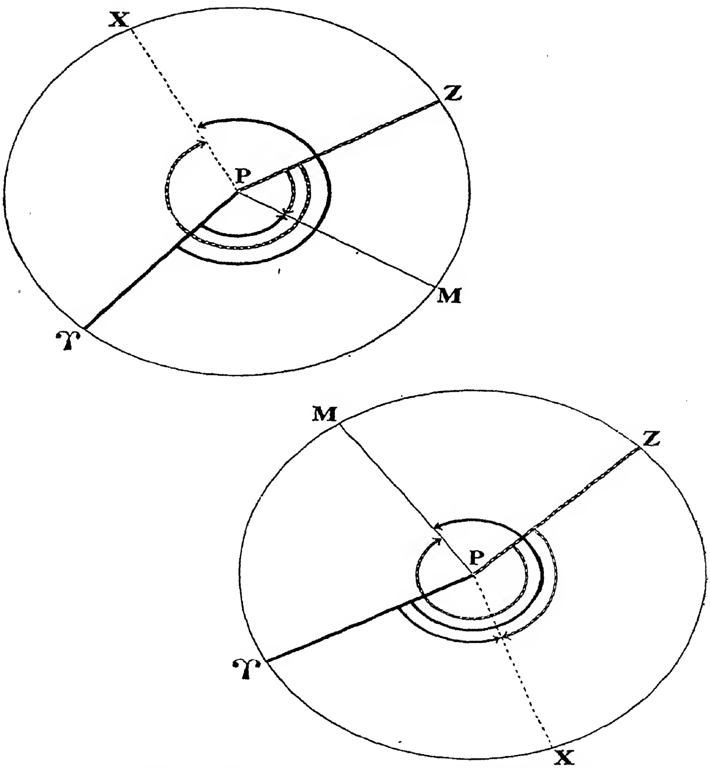


FIGURE 88a. FIGURE 88b.

The Hour Angle of a Star. The formula giving the hour angle may therefore be adjusted in terms of R. It then becomes :

—and the procedure for finding the hour angle may be summarised thus :

(1) Find the G.D.
(2) Apply the error to the D.W.T. and find G.M.T.
(3) From the *Nautical Almanac* take out R for this G.M.T., interpolating if necessary, and look up the star's right ascension, which is given in the extracts from the star tables printed at the beginning of the tables for each month or in the star tables themselves.
(4) Apply the longitude of the place to G.M.T. and obtain L.M.T.
(5) Add R to L.M.T. and so obtain R.A.M.
(6) Subtract R.A.✳ from R.A.M. and so obtain the H.A.✳.

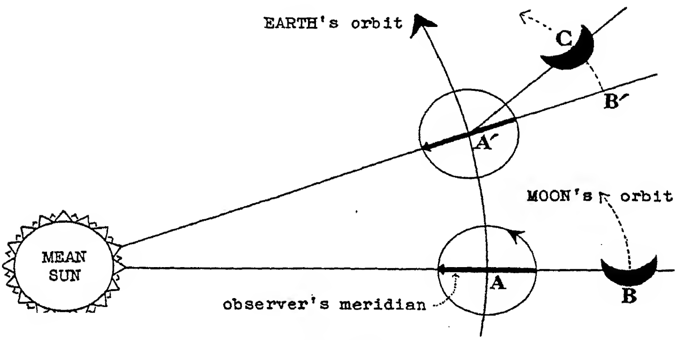


FIGURE 89.

The Hour Angle of the Moon. If two successive transits of the Moon were observed across the same meridian, the interval between them would be a lunar day, and, since the Moon is itself describing an orbit about the Earth in the same direction as the Earth's spin, this interval would be longer than the mean solar day. (Figure 89.)

AA' is a measure of the mean solar day, but while the Earth has moved from A to A', the Moon has reached C, and the Earth will have to turn through a further angle approximately equal to $B'A'C$ before the Moon is on the observer's meridian. The average time taken to turn this extra angle is 50^m.

The units derived from the lunar day are lunar units, but, for the same reason that it is not necessary to work in sidereal units in order to find the hour angle of a star, it is not necessary to work in lunar units to find the hour angle of the Moon. The right ascension of the Moon is given in the *Nautical Almanac* for every two hours throughout the year, and by careful interpolation it can be found for

any instant of G.M.T. The formula giving the hour angle of a star can therefore be used for finding the hour angle of the Moon.

If, instead of Achernar in the previous example, the Moon had been observed, its hour angle would have been :

Z.T.	2035	
Zone	0	
G.D.	2035 3rd April 1937	
	h m s	R.A.☾ h m s
D.W.T.	8 34 35	18 42 39
Error fast	04	+1 14·5
G.M.T.	20 34 31 3rd April	18 43 53·5
Long. W.	13 04	
L.M.T.	20 21 27	
R	12 47 00	
R.A.M.	9 08 27	
R.A.☾	18 43 53	
H.A.☾	14 24 34	

The Lunation or Lunar Month. This is the interval between two successive new moons, and it is important in tidal work.

The Moon makes one complete revolution about the Earth in $27\frac{1}{3}$ mean solar days, but if the Moon were new at the beginning of this period, when the Earth is at *A* in figure 90, it would not be new at the end because it would not lie in a straight line with the Sun and the Earth, which is now at *B*. Before it does that, it must move along its orbit round the Earth, and while this is happening the Earth continues along its own orbit round the Sun to a position *C*. Approximately $2\frac{1}{6}$ mean solar days elapse before the Moon is again new, and a lunation or lunar month is therefore about $(27\frac{1}{3}+2\frac{1}{6})$ or $29\frac{1}{2}$ mean solar days.

In this period the Moon must cross the meridian once less than the Sun, and this fact establishes the 50-minute difference between the mean solar day and the lunar day because :

$$28\tfrac{1}{2} \text{ lunar days} \equiv 29\tfrac{1}{2} \text{ mean solar days}$$

i.e.
$$1 \text{ lunar day} \equiv \frac{29\frac{1}{2}}{28\frac{1}{2}} \text{ mean solar days}$$

$$\text{or} \quad 1^{d}0^{h}50^{m}$$

The Hour Angle of a Planet. The planets belong to the solar system, and their motion in the celestial sphere is therefore different from that of a star. For the purpose of finding its hour angle, however, a planet can be treated as a star just as the Moon has been

treated as a star. The right ascensions and declinations of the four planets sufficiently bright to be observed—Venus, Mars, Jupiter and Saturn—are given in the *Nautical Almanac* for each day.

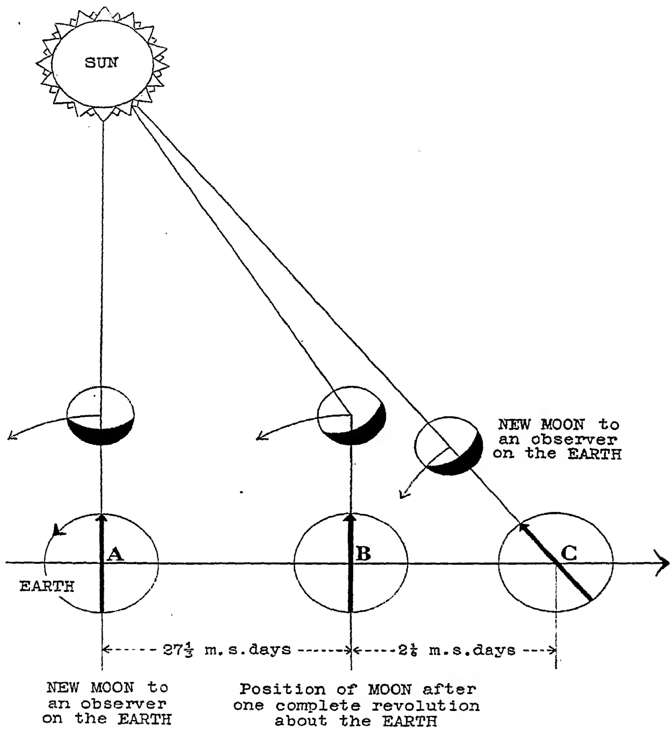


FIGURE 90.

The formula L.M.T.+R=H.A.+R.A., it should be noted, applies to any heavenly body. It is used, indirectly, for giving the hour angle of the Sun. The quantity E is simply (R−R.A.⊙).

CHAPTER XII

THE G.H.A. ALMANAC

In this type of almanac, the actual hour angle of the heavenly body, measured from the Greenwich meridian, is tabulated in units of arc at convenient intervals of Greenwich mean time.

As figure 91 shows, in order to find the hour angle measured from any meridian, it is necessary to do no more than apply the longitude of that meridian to the quantity taken from the almanac, according to the ordinary rule: longitude west, Greenwich time best; longitude east, Greenwich time least. West longitude is always subtracted from the Greenwich hour angle, to which 360° is

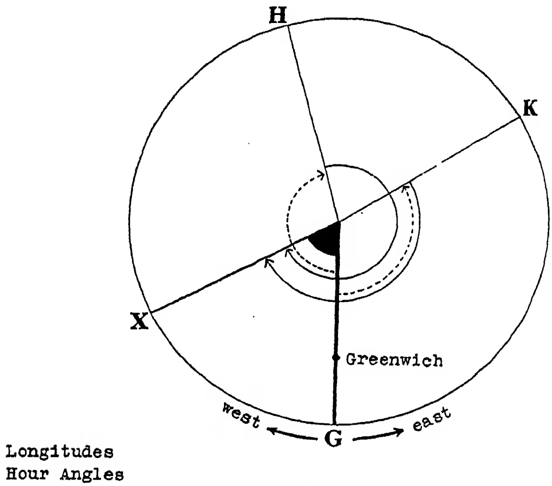


FIGURE 91.

added if necessary. East longitude is always added, and 360° is subtracted from the resulting sum if necessary.

If, for example, the Greenwich hour angle of a heavenly body is found from the almanac to be 62°39′, and the longitude of the meridian through *H* is 164°47′W., and that of the meridian through *K* is 121°13′E., then:

At H

G.H.A.	62°39′
	360°
	422°39′
Long. W.	164°47′
Local H.A.	257°52′

At K

G.H.A.	62°39′
Long. E.	121°13′
Local H.A.	183°52′

The Greenwich hour angles of the Sun, the Moon and the four navigational planets—Venus, Mars, Jupiter and Saturn—can be given directly and easily, but it is not so easy to give the Greenwich hour angles of all the stars likely to be observed by navigators, because a direct tabulation at short intervals takes up far too much space, and if the space is cut down by increasing the intervals, elaborate interpolation tables are necessary. These disadvantages, however, can be avoided by tabulating the Greenwich hour angle of one 'standard star'—the First Point of Aries—and then adding to the angle thus obtained a quantity that is constant for a particular star. This method is adopted in the *Air Almanac.*

The Sidereal Hour Angle. (S.H.A.) The sidereal hour angle of a heavenly body is the angle between the meridian of the First Point

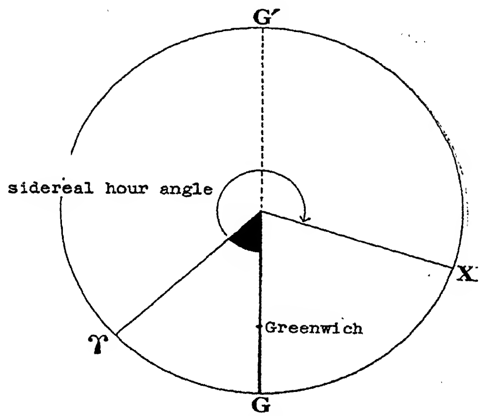


FIGURE 92.

of Aries and the meridian of the heavenly body, measured westward from the First Point of Aries. The sidereal hour angle of a heavenly body is thus 360° minus the heavenly body's right ascension.

In figure 92, the Greenwich hour angle of ♈ is represented by $G♈$, the right ascension of X by $♈GX$, and the sidereal hour angle by $♈G'X$. Clearly $G♈G'X$ is equal to $G♈$ plus $♈G'X$. That is, the sidereal hour angle is always added to the Greenwich hour angle of Aries, and the result is the Greenwich hour angle of the heavenly body.

Once the Greenwich hour angle of a star has been found, the hour angle measured from any other meridian is obtained by adding or subtracting the longitude of that meridian. If, for example, the star is Betelgeuse and the meridian 74°39′W., its local hour angle at

16^h20^m G.M.T. on the 1st October 1937 is obtained thus (see the extract from the *Air Almanac* given on page 259 of the Appendix) :

G.H.A.♈	254°59′
S.H.A.✳	272°03′
G.H.A.✳	527°02′
Long. W.	74°39′
L.H.A.✳	452°23′
	360°
	92°23′

Accuracy of Almanacs Tabulated to One Minute of Arc. For surface navigation it is accepted that quantities should be tabulated to 0′·1. The rounding off of a quantity to 1′ introduces a possible error of 0′·5, and the addition of two such quantities—the G.H.A. of the Sun for the hour and the correction for minutes and seconds, for example—doubles this possible error. The error in the G.H.A. of a heavenly body thus depends on the number of quantities that form it. A similar error occurs in the declination, but this will not, in general, be greater than 0′·5 because the declination can be taken from the almanac without interpolation. The chance of meeting the maximum possible errors in all quantities simultaneously is, however, so unlikely that it can be disregarded. Also an error even as large as 1′ in G.H.A. and declination, though introducing a maximum error of just over 5′ in the observed position when the position lines cut at 30°, introduces one of only 2′ when they cut at 90°. An almanac tabulating quantities to 1′ can therefore be used for surface navigation in open waters without fear of introducing any serious error, but it should not be used near land, except with caution, because the error resulting from its use is not the only error to be expected in the observed position. There are those resulting from errors in observation and logging, and from the error inherent in the method by which the sights are worked, and near land these unavoidable errors should not be added to unnecessarily.

Interpolation Tables. When the hour angle is required at a G.M.T. lying between the times for which quantities are tabulated, interpolation must be carried out between the quantities nearest the given time, and for this purpose special interpolation tables are provided for each type of heavenly body. This is necessary because the Sun, the Moon, the stars and the planets have different motions.

The motion of the Sun, for example, is 15° per hour, if changes in the equation of time during that period are neglected, but the Moon's motion varies appreciably and is roughly half a degree per hour less than the Sun's. A star's motion is constant and approximately equal to 15°02′·5 per hour. The motion of the planets lies between about 14°59′ and 15°04′ per hour.

These figures show that the construction of interpolation tables for an almanac giving the Greenwich hour angle in arc is a matter of some complexity, involving relatively large quantities. It is therefore convenient to have, at times, more than one table, and to arrange these tables as *critical* tables which require no interpolation between the quantities taken from them. This arrangement is adopted in the *Air Almanac.*

Critical Tables. In an ordinary table, values of the quantity required (which is known as the *respondent*) correspond to particular values of the argument at convenient and equal intervals of that argument. Interpolation is then necessary in order to arrive at the value of the respondent for an intermediate value of the argument, and even when the interval of tabulation is so small that successive values of the respondent differ by only 1 or 2 in the last figure given, the interpolated value may be in error by a whole unit. But in a critical table, the argument is divided into intervals so chosen that successive intervals correspond to successive integral values of the respondent. For any value of the argument inside one of these intervals, the respondent is found without interpolation and with less than half a unit in the last figure given. The *critical* values of the argument correspond to half-way values of the respondent and are always chosen so that, when the argument is equal to one of the critical values, the value of the respondent corresponding to the *previous* interval is to be used. The rule to be followed is therefore: *when the values of the argument are critical, ascend.*

The tables reproduced on page 261 of the Appendix—they are extracts from those printed on the inside of the front cover of the actual almanac—show that 01^m29^s and 05^m57^s are critical values of the argument, and therefore correspond to increments of 0°37′ and 1°29′ respectively, both these increments being the upper of the possible values.

The Hour Angle of the Sun obtained from the *Air Almanac.* The Greenwich hour angle of the Sun at any instant is found by adding the Greenwich hour angle for the preceding hour, taken directly from the tabulated data, to the correction for the Sun's motion in the odd minutes and seconds, taken from the interpolation table for the Sun. Thus, the local hour angle of the Sun at $16^h25^m31^s$ G.M.T. on the 1st October 1937 in longitude 86°34′W. is given by:

G.H.A. (16^h)	62°34′
Table (25^m31^s)	6°23′
G.H.A. ($16^h25^m31^s$)	68°57′
	360°
	428°57′
Longitude W.	86°34′
L.H.A. ($16^h25^m31^s$)	342°23′

The Hour Angle of the Moon from the *Air Almanac*. The Moon's interpolation table is based on a mean motion of 14°20′ per hour in order to keep all corrections positive, and the Greenwich hour angle of the Moon is the sum of three quantities : the Greenwich hour angle for the preceding hour taken directly from the tabulated data, and the corrections for the Moon's motion in the odd minutes and seconds which are obtained from the interpolation table and from the table of proportional parts headed ' P.PS.'. The maximum value of this last term is 19. It is always positive, and it should be interpolated (where necessary) to the minutes of G.M.T.

The local hour angle of the Moon at $17^h23^m37^s$ G.M.T. on the 1st October 1937 in longitude 23°29′E., for example, is given by:

G.H.A. (17^h)	110°17′
Table (23^m37^s)	5°39′
P.PS. (24^m)	05′
G.H.A. ($17^h23^m37^s$)	116°01′
Longitude E.	23°29′
L.H.A. ($17^h23^m37^s$)	139°30′

The Hour Angle of a Planet from the *Air Almanac*. The interval of tabulation for the planets is 6^h, but the method of taking out the Greenwich hour angle of a planet is exactly analogous to that for the Moon. The main interpolation table to be used, however, is that for the Sun.

The Hour Angle of a Star from the *Air Almanac*. The Greenwich hour angle of the First Point of Aries is tabulated for intervals of 10 minutes, and the Greenwich hour angle of any star is therefore the sum of three components : the sidereal hour angle of the star, the Greenwich hour angle of the First Point of Aries for days, hours, and tens of minutes of G.M.T., and the correction for the remaining minutes and seconds within the interval of 10 minutes.

The local hour angle of Betelgeuse at $18^h27^m42^s$ G.M.T. on the 1st October 1937 in longitude 72°23′E., for example, is given by :

S.H.A. Betelgeuse	272°03′
G.H.A. ♈ (18^h20^m)	285°04′
Increment for 7^m42^s	1°56′
G.H.A. Betelgeuse	559°03′
Longitude E.	72°23′
	631°26′
	360°
L.H.A. ($18^h27^m42^s$)	271°26′

Full explanations of these methods, and the procedure to be followed in taking out the declination, are given in the *Air Almanac*.

CHAPTER XIII

THE SOLUTION OF THE SPHERICAL TRIANGLE *PZX*

The actual solution of the spherical triangle—the calculation, that is, of the remaining angles and sides when certain of them are given—offers the same trigonometrical problem whether the triangle itself is *PFT* on the Earth's surface or *PZX* on the celestial sphere.

Figure 93 shows that the substitution of *Z* for *F* and *X* for *T* suffices to make the '*PFT*' formulæ suitable for solving the triangle *PZX*.

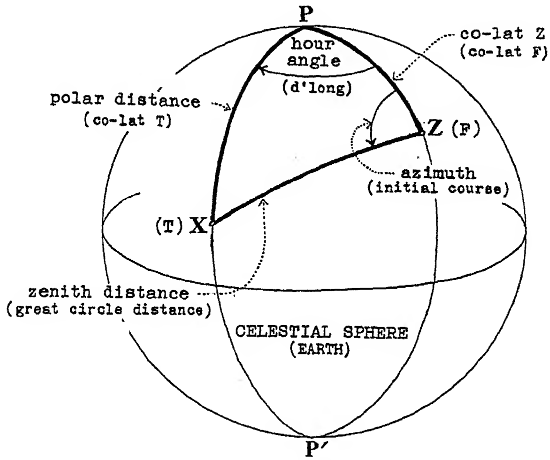


FIGURE 93.

The two main problems are :

(1) to find the zenith distance when the hour angle, latitude and declination are known.

(2) to find the azimuth when the latitude, declination and zenith distance are known.

As in the solutions of *PFT*, the natural haversine formula suffices for (1) and the 'half log haversine' formula for (2).

The haversine formula adjusted for *PZX* is :

$$\text{hav } ZX = \text{hav } (PX \sim PZ) + \sin PX \sin PZ \text{ hav } ZPX$$

Written thus, it gives $\underline{ZX}$, the zenith distance or side opposite the hour angle $Z\underline{P}X$.

When written :

$$\text{hav } PX = \text{hav } (PZ \sim ZX) + \sin PZ \sin ZX \text{ hav } PZX$$

—it gives $\underline{PX}$, the polar distance or side opposite the azimuth $P\underline{Z}X$.

Modification of the Haversine Formula. It was noted on page 34 of Chapter IV that the haversine formula could be simplified. This is so because :

$$PX = 90° \pm \text{declination}$$
$$PZ = 90° - \text{latitude}$$

Hence it follows that :

$$\sin PX = \sin (90° \pm \text{dec.}) = \cos (\text{dec.})$$
$$\sin PZ = \sin (90° - \text{lat.}) = \cos (\text{lat.})$$

Also :

$$(PX \sim PZ) = (90° \pm \text{dec.}) \sim (90° - \text{lat.})$$

—so that :

$$\text{hav } (PX \sim PZ) = \text{hav } (\text{lat.} \sim \text{dec.})$$

—since the left-hand side of the equality is always the haversine of a difference between PX and PZ, never the haversine of their sum, and the two quantities, each 90°, on the right-hand side therefore cancel, leaving either (dec.—lat.) or (lat.—dec.) or (lat.+dec.). Clearly the latitude and declination are added if they have opposite names. Otherwise the smaller is subtracted from the larger.

The haversine formula thus becomes :

$$\text{hav } ZX = \text{hav } (\text{lat.} \sim \text{dec.}) + \cos (\text{lat.}) \cos (\text{dec.}) \text{ hav } (\text{hour angle})$$

—and this is the form in which it is used when the zenith distance is required.

The 'Half Log Haversine' Formula. The 'half log haversine' formula applied to the spherical triangle PZX is :

$$\text{hav } PZX = \text{cosec } PZ \text{ cosec } ZX \sqrt{\text{hav } [PX + (PZ \sim ZX)] \text{ hav } [PX - (PZ \sim ZX)]}$$

Written thus, it gives the azimuth $P\underline{Z}X$, which is the angle between $P\underline{Z}$ and $\underline{Z}X$.

When written :

$$\text{hav } ZPX = \text{cosec } PZ \text{ cosec } PX \sqrt{\text{hav } [ZX + (PZ \sim PX)] \text{ hav } [ZX - (PZ \sim PX)]}$$

—it gives the hour angle $Z\underline{P}X$, which is the angle between $Z\underline{P}$ and $\underline{P}X$.

If this sequence of letters is borne in mind, there should be no difficulty in remembering the formulæ and the results they give.

OBSERVER IN NORTH LATITUDE

To Calculate the Zenith Distance. Suppose that the observer's latitude is 50°N., and that a star's declination is 20°S., the hour angle being $2^h 30^m$.

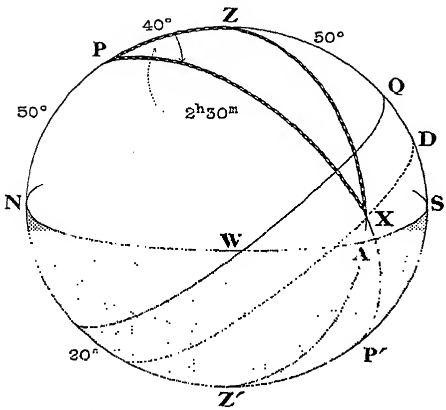
The pole above the horizon is the north pole. The hour angle shows that the star is in the western half of the celestial sphere. The north point N therefore lies to the left in the figure on the plane of the meridian. (Figures 94a and 94b.)

The haversine formula arranged to give the zenith distance is:

$$\text{hav } ZX = \text{hav } (l \sim d) + \cos l \cos d \text{ hav (H.A.)}$$

Hence:

H.A.	2^h30^m	log hav (H.A.)	9·014	20
lat.	50°N.	log cos l	9·808	07
dec.	20°S.	log cos d	9·972	99
=	70°	(log hav)	8·795	26
		(hav)	·062	41
		hav ($l \sim d$)	·328	99
		hav ZX	·391	40



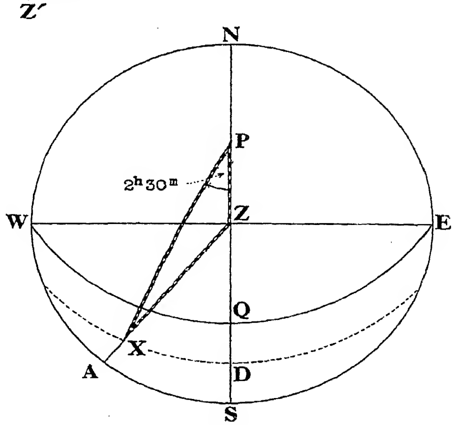


FIGURE 94a. FIGURE 94b.

The calculated zenith distance, or C.Z.D., is therefore 77°27′·3. If required, the calculated altitude is found at once by subtracting the C.Z.D. from 90°.

To Calculate the Azimuth. Suppose that the circumstances are those of the last example. In the spherical triangle PZX the three sides are now known, and the 'half log haversine' formula suffices to give any angle.

Since the angle at Z is required, the particular formula starts with the sides about the angle at Z: PZ and ZX. Thus:

hav PZX

$=$cosec PZ cosec . $\overline{[PX+(PZ\sim ZX)] \text{ hav } [PX-(PZ\sim ZX)]}$

PZ	$= 40°00'{\cdot}0$	log cosec $40°00'{\cdot}0$	0·191 93
ZX	$= 77°27'{\cdot}3$	log cosec $77°27'{\cdot}3$	0·010 49
	$= 37°27'{\cdot}3$		
PX	$=110°00'{\cdot}0$		
$PX+(PZ\sim$)		$\frac{1}{2}$ log hav $147°27'{\cdot}3$	4·982 25
$PX-(PZ\sim ZX)$	$= 72°32'{\cdot}7$	$\frac{1}{2}$ log hav $72°32'{\cdot}7$	4·772 05
		log hav PZX	9·956 72

The angle PZX is therefore $144°07'{\cdot}5$.

Since the pole above the horizon is the north pole, and the hour angle lies between 0^h and 12^h, the azimuth, to the nearest degree, is N.144°W.

The true bearing is (360°—144°); that is, 216°.

To Calculate the Declination. Suppose that the true bearing of a star is 140° from an observer in 50°N., and that the zenith distance is 70°.

The pole above the horizon is the north pole. The true bearing of 140° shows that the star is in the eastern half of the celestial sphere. The north point N therefore lies to the right in the figure on the plane of the meridian.

In the spherical triangle PZX—figures 95a and 95b—two sides and the angle between them are known:

$$90°-\text{latitude}=40°$$
$$ZX= 70°$$
$$\angle PZX=$$

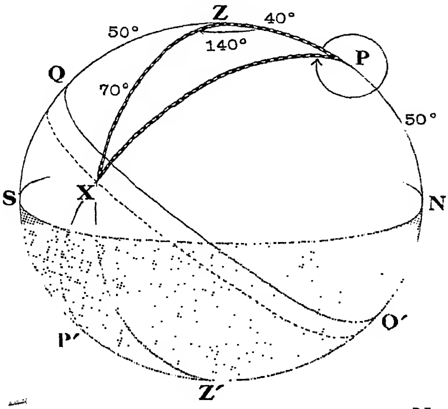
Hence:

$\angle PZX$	$=140°$	log hav PZX	9·945 97
ZX	$= 70°$	log sin ZX	9·972 99
PZ	$= 40°$	log sin PZ	9·808 07
$ZX\sim PZ=$	$30°$	(log hav)	9·727 03
		(hav)	·533 36
		hav $(ZX\sim PZ)$	·066 99
		hav PX	·600 35

PX is therefore $101°34'{\cdot}7$, and the declination is $11°34'{\cdot}7$S.

For the sake of clarity the exact designation has been put against each logarithm in the preceding examples. This, however,

is not done in practice, and in the examples that follow the logarithms are given without further explanation.



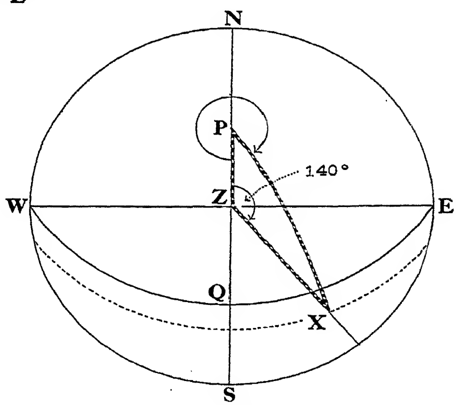


FIGURE 95a. FIGURE 95b.

To Calculate the Hour Angle. The hour angle of the star in the last example can be found by using the formula :

$$\text{hav } ZPX = \text{cosec } PZ \text{ cosec } PX \sqrt{\text{hav } [ZX+(PZ \sim PX)] \text{ hav } [ZX-(PZ\sim PX)]}$$

PZ	$= 40°00'\cdot0$	0·191 93
PX	$=101°34'\cdot7$	0·008 93
$PZ\sim PX$	$= 61°34'\cdot7$	
ZX	$= 70°00'\cdot0$	
$ZX+(PZ\sim PX)$	$=131°34'\cdot7$	4·960 02
$ZX-(PZ\sim PX)$	$= 8°25'\cdot3$	3·865 85
		9·026 73

The angle ZPX is therefore $2^h32^m16^s$.

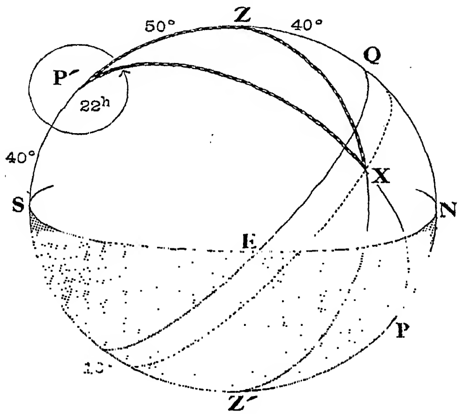
But the hour angle, being easterly, lies between 12^h and 24^h. Hence the angle required is :

$$(24^h - 2^h32^m16^s)$$

i.e. $$21^h27^m44^s$$

OBSERVER IN SOUTH LATITUDE

Since the hour angle of a heavenly body is essentially an angle between two meridians, it can be measured either at the north pole



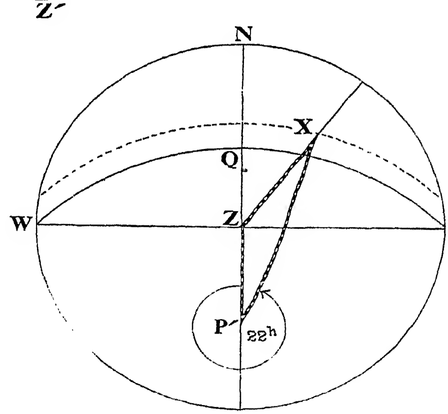


FIGURE 96a. FIGURE 96b.

or at the south pole. The methods of solving the spherical triangle PZX therefore apply to the spherical triangle $P'ZX$, where P' is the south pole.

When, for example, Z lies in one hemisphere and X in the other, the zenith distance can be found equally well from either triangle.

To Calculate the Zenith Distance. Suppose that the observer's latitude is 40°S., and that a star's declination is 10°N., the hour angle being 22^h.

The pole above the horizon is the south pole. The hour angle shows that the star is in the eastern half of the celestial sphere. The south point S therefore lies to the left in the figure on the plane of the meridian. (Figures 96a and 96b.)

H.A. $=22^h$	8·825	99
lat. $=40°$S.	9·884	25
dec. $=10°$N.	9·993	35
$l \sim d = 50°$	8·703	59
	·050	54
	·178	61
	·229	15

The C.Z.D. is therefore 57°12′·0.

To Calculate the Azimuth.

$P'Z$	$= 50°00'$	0·115	75
ZX	$= 57°12'$	0·075	43
$P'Z \sim ZX$	$= 7°12'$		
$P'X$	$= 100°00'$		
$P'X + (P'Z \sim ZX)$	$= 107°12'$	4·905	74
$P'X - (P'Z \sim ZX)$	$= 92°48'$	4·859	84
		9·956	76

The angle $P'ZX$ is therefore 144°08′·5; the azimuth is S.144°E., and the true bearing 036°.

CHAPTER XIV

THE ASTRONOMICAL POSITION LINE

On the ability to solve the triangle PZX when the necessary information is given depends the whole theory of position-finding at sea, because it enables the navigator to link his D.R. position (which is approximate) with the observed altitude (which is theoretically exact) and thus obtain a position line when, as usually happens, the distance between his D.R. position and the geographical position of the heavenly body is large.

The Position Circle. When an observer measures the altitude of a heavenly body he obtains from it, by correction and subtraction

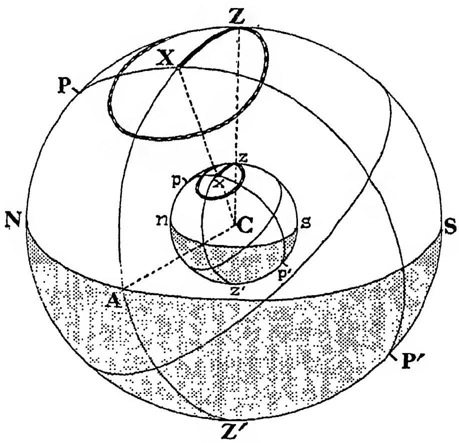


FIGURE 97.

from 90°, a true zenith distance—ZX in figure 97. Z, therefore, might be any point on a small circle of radius ZX and centre X.

On the Earth the observer's position, z, lies on the circumference of a small circle, the centre of which is the heavenly body's geographical position. The radius of this circle is also the true zenith distance, zx, and since it is now measured on the surface of the Earth, it can be expressed in nautical miles.

This small circle is known as a *position circle*.

The astronomical position line is the small arc of this position circle on which the observer or navigator discovers his position to be.

If zx is very small, some twenty miles or so, the geographical position can be plotted on the chart and the actual circle drawn without loss of accuracy, but in general zx will be large, of the order 1,000 miles, and the geographical position will seldom be on the chart that the navigator is using for keeping his reckoning. The part of the position circle that concerns the navigator must therefore be found by methods that confine the plotting they involve to the neighbourhood of the ship's actual position.

There are two methods in common use that do this :

(1) the Marc St. Hilaire or ' intercept ' method.

(2) the ' longitude ' method.

THE MARC ST. HILAIRE OR INTERCEPT METHOD

In this method the navigator chooses any position in the neighbourhood of the position where he thinks the ship is. His D.R. or estimated position is in some ways the most convenient.

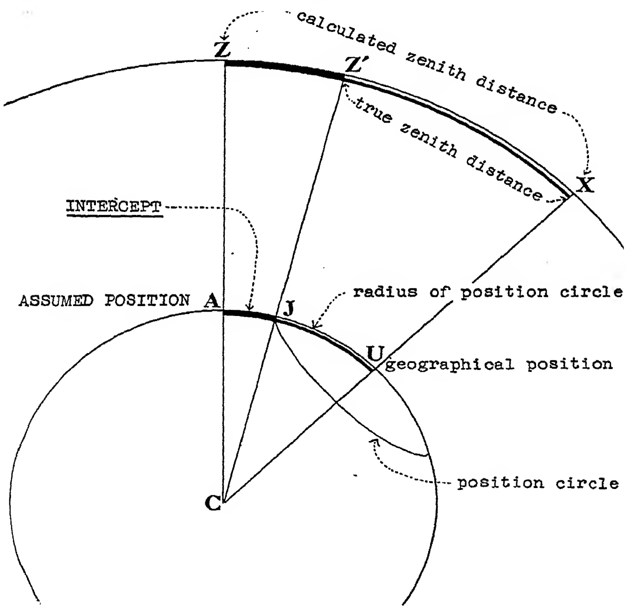


FIGURE 98.

In figure 98, A is this D.R. position and Z its zenith. The geographical position of the heavenly body X is U. AU is an arc of the great circle passing through A and U. The radius of the position circle found from an observed altitude of X is JU, and this will not, as a rule, be equal to AU. The difference, AJ, is called the *intercept*.

Figure 99 shows these positions, *A*, *J* and *U*. in relation to the pole *P*, and it is at once clear that the distance *AU* (which is equal in angular measurement to *ZX* in the corresponding triangle on the celestial sphere) is simply the *calculated zenith distance* found by solving the spherical triangle *PAU* or *PZX*. *UJ* is the *true zenith distance*, and the position of *J* is decided by the azimuth of the heavenly body, which is the angle *PAU* or *PZX*.

If, for example, the C.Z.D. is 50°, the navigator knows that his D.R. position (or whatever position he chooses for working the sight) is 3,000′ from the geographical position. If, at the same time, his true zenith distance is 49°55′, he also knows that his own position is 2,995′ from the geographical position. That is, his own position is 5′ nearer the geographical position than the position from which he worked the sight. Also, his position lies on the circumference of

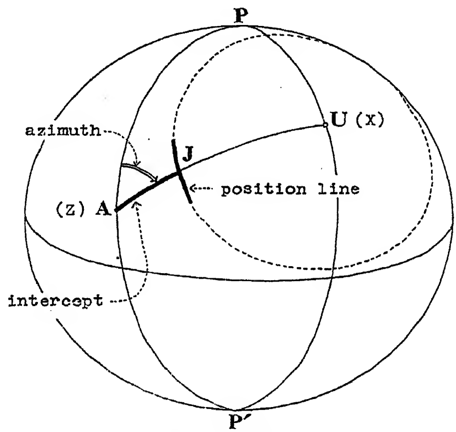


FIGURE 99.

the position circle. If, then, a line of bearing is drawn on the chart from the D.R. and a length equivalent to 5′ marked off in the direction of the heavenly body, the point obtained will be the point *J* (figure 100) and the line of bearing will coincide with the radius of the position circle for a short distance.

J is therefore one point that is the correct distance from the geographical position, but it is not necessarily the ship's position. That lies somewhere on the circumference of the position circle in the neighbourhood of *J*.

Since the circumference of a circle at any point is at right-angles to this radius at this point, the small arc that is the position line can be drawn as a straight line through *J* at right-angles to the line of bearing.

The Intercept. The essential feature of the intercept method is thus a comparison between the ship's known distance from the

geographical position and the calculated distance of some arbitrary position (usually the D.R.) from that same geographical position. The first distance is the T.Z.D. and the second the C.Z.D.

If the T.Z.D. is greater than the C.Z.D., the ship's actual position is farther from the geographical position than the position arbitrarily chosen, and the intercept is therefore *away*.

If the T.Z.D. is less than the C.Z.D., the ship's actual position is nearer that geographical position, and the intercept is *towards*.

Assumptions made when the Position Line is Drawn. There are three assumptions made when a position line is drawn as a straight line on a Mercator chart.

(1) The bearing of the geographical position is the same at all points in the neighbourhood of *J*.

(2) The intercept, which is laid off as a straight line and therefore represents a rhumb line, coincides with the great circle that is the actual line of bearing.

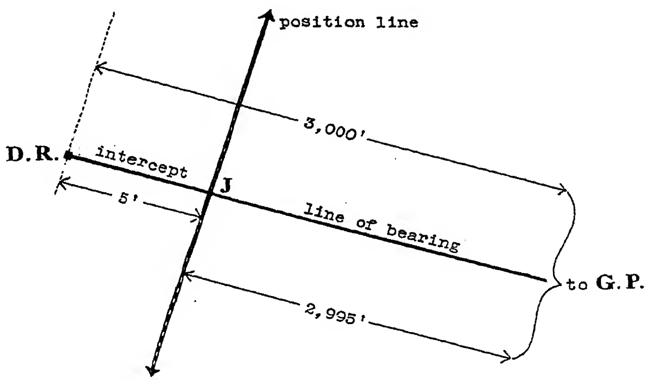


FIGURE 100.

(3) The position line itself, which is also laid off as a straight line and therefore represents a rhumb line, coincides with the arc of the position circle over the small length shown on the chart.

All three assumptions are justified in normal circumstances because the error introduced is negligible. Only when the altitude is so large that the position circle can be drawn on the chart as a circle are these assumptions inadmissible. The procedure described in Chapter IV of Volume III must then be followed. The first assumption is discussed separately in Chapter X of that volume.

Procedure for Finding the Intercept. The steps by which the sight is worked are :

(1) Find the Greenwich Date.

(2) Find the G.M.T. and the hour angle of the heavenly body by the methods described in Chapters X and XI.

(3) From the spherical triangle *PZX* find the calculated zenith distance and the azimuth. (Chapter XIII.)

(4) Correct the observed altitude and obtain the true zenith distance. (Chapter VIII.)

(5) From the C.Z.D. and the T.Z.D. find the intercept.

Example of a Sun Sight. *At Z.T.* 0750(+2), 3*rd April* 1937, *the D.R. was* 34°31′*N*., 28°36′*W*., *and the deck watch showed* $9^h51^m33^s$ *when the sextant altitude was* 26°18′·0. *The deck watch was* 31^s *slow on G.M.T.; the index error was* —2′·6, *and the height of eye* 32 *feet. Draw the position line.*

			Sun's Declination
Z.T.	0750 3rd April		5°11′·7N.
Zone	+2		+1′·8
G.D.	0950 3rd April		5°13′·5N.
	h m s		
D.W.	9 51 33		
Error slow	31		E
			h m s
G.M.T.	9 52 04 3rd April		11 56 32·2
Long. W.	1 54 24		+1·4
L.M.T.	7 57 40		11 56 33·6
E	11 56 34		
H.A.T.S.	19 54 14	9·416 60	
Lat. N.	34°31′·0	9·915 91	
Dec. N.	5°13′·5	9·998 20	
$l \sim d$	29°17′·5	9·330 71	
		·214 15	
		·063 93	
		·278 08	

C.Z.D.=63°39′·1

Sext. Alt.	26°18′·0	
I.E.	—2′·6	
Obs. Alt.	26°15′·4	
Corr^n.	+8′·6	
True Alt.	26°24′·0	
	90°	
T.Z.D.	63°36′·0	
C.Z.D.	63°39′·1	
Intercept		3′·1 towards

Calculation of the azimuth :

PZ	$55°29'·0$	0·084	09
ZX	$63°39'·1$	0·047	63
	$8°10'·1$		
PX	$84°46'·5$		
$+$	$92°56'·6$	4·860	36
$-$	$76°36'·4$	4·792	27
$\angle PZX$	$=102°32'·9$	9·784	35

Since the Sun's hour angle is greater than 12^h the Sun itself must lie to the east of the observer, and its azimuth must be N.$102\frac{1}{2}$°E.

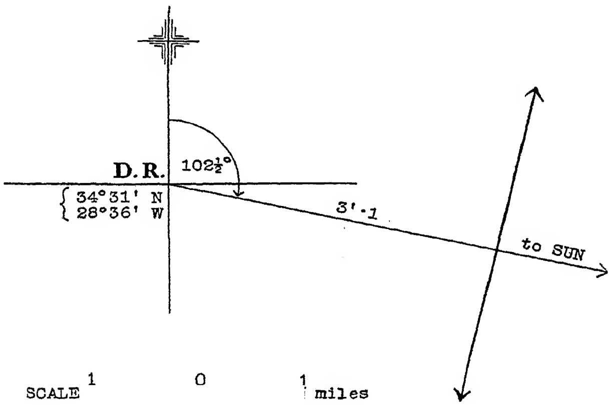


FIGURE 101.

Figure 101 shows how the results of these calculations are plotted on the chart.

Use of the Astronomical Position Line. The position line thus obtained can be used exactly as if it had been derived from the observation of a terrestrial object. (Chapter VI.) The same points should be noted about it.

(1) It does not give the navigator his position, but tells him that his position lies somewhere on it.

(2) Two position lines are necessary to give the navigator's actual position, which will be at their point of intersection.

(3) If an appreciable time elapses before the second position line is obtained, the first must be transferred a distance equal to the ship's run in that time.

Example of a Star Sight. *At Z.T.* 2110(—8), *3rd April* 1937, *the D.R. was* 21°15'*N.*, 115°38'*E.*, *and the deck watch showed* $1^h10^m43^s$

when the sextant altitude of Arcturus was 26°25′·2. *The deck watch was* 15ˢ *slow on G.M.T.; the index error was* +2′·3 *and the height of eye* 28 *feet. Draw the position line.*

		R
		h m s
Z.T.	2110	12 45 35·7
Zone	−8	11·7
G.D.	1310 3rd April	12 45 47·4

	h m s			
D.W.T.	1 10 43			
Error slow	15			*Arcturus*
G.M.T.	13 10 58 3rd April		R.A.	$14^h12^m50^s$
Long. E.	7 42 32		Dec.	19°30′·3N.
L.M.T.	20 53 30			
R	12 45 47			
R.A.M.	9 39 17			
R.A.	14 12 50			
H.A.	19 26 27	9·499 46		
Lat. N.	21°15′·0	9·969 42		
Dec. N.	19°30′·3	9·974 33		
$l \sim d$	1°44′·7	9·443 21		
		·277 47		
		·000 23		
		·277 70		

C.Z.D. = 63°36′·1

Sext. Alt.	26°25′·2
I.E.	+2′·3
Obs. Alt.	26°27′·5
Corrⁿ.	−7′·2
True Alt.	26°20′·3
T.Z.D.	63°39′·7
C.Z.D.	63°36′·1
Intercept	3′·6 away

Azimuth tables show that the star's azimuth is N.78°E.

Figure 102 shows the intercept as it would appear if drawn on a Mercator chart.

Azimuth Tables. The use of tables wherein the triangle *PZX* is solved and the azimuth found for successive values of the latitude,

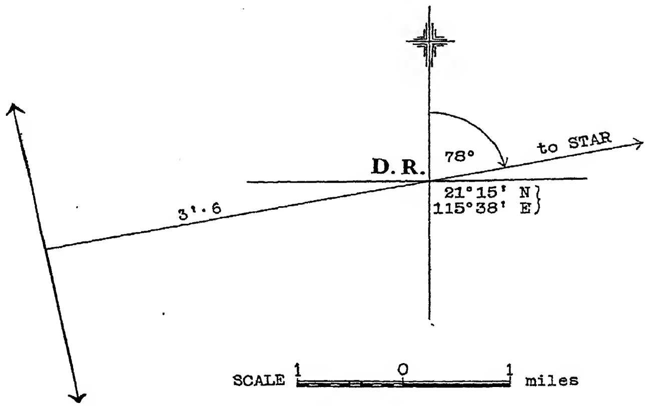


FIGURE 102.

declination and the hour angle, considerably reduces the labour of calculating the azimuth.

To be able to use these tables—especially the older patterns of Davis and Burdwood—it is essential to understand that they give azimuths east and west of the meridian for hour angles measured east and west of it.

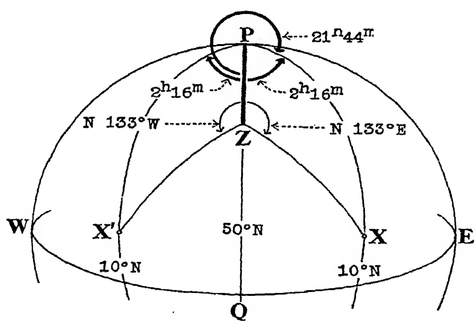


FIGURE 103.

For example, when the latitude and declination are 50°N. and 10°N. respectively, these tables give 133° as the value of the angles *PZX* and *PZX'* in both the spherical triangles *PZX* and *PZX'*, the *smaller angle at* P *being* 2^h16^m *in each triangle.* (Figure 103.) But

in the triangle *PZX'*, the hour angle is measured westward, and in *PZX* it is measured eastward. (Actually, when measured westward, it is ($24^h—2^h16^m$) or 21^h44^m.) The azimuth is therefore N.133°W., for the westerly hour angle, and N.133°E. for the other.

For heavenly bodies with declinations less than 24° (in the so-called Sun Tables) the argument for the triangle *PZX'* is called either *Apparent Time* 2^h16^m *P.M.* or *Star's H.A.*, and that for the triangle *PZX* is called *Apparent Time* 9^h44^m *A.M.* For heavenly bodies with declinations greater than 24° (in the so-called Star Tables) the argument is given for one triangle only as *H.A.* 2^h16^m.

To avoid confusion, therefore, it should be remembered that the tables can be used for any heavenly body, Sun, star, Moon or planet ; and that since the azimuth depends for its name upon the smaller angle at the pole between the observer's meridian and the meridian of the heavenly body, the tables can be most conveniently entered for this smaller angle, obtained if necessary by subtracting the true hour angle from 24^h. The A.M. and P.M. arguments can then be ignored.

The following table, which illustrates the points referred to in the above example, is an extract from Burdwood's Sun Tables. It also shows that if the latitude is 50°20′N., say, and the declination is 10°37′N., some awkward interpolation must be carried out in order to obtain the azimuth with any accuracy.

LATITUDE 50°

Apparent Time A.M.		DECLINATION—same Name as—LATITUDE							Apparent Time P.M.	
		0°	1°	2°	...	9°	10°	11°		
h	m					° ′	° ′	° ′	h	m
X	0	—	—	—		138 20	137 44	137 7	11	0
IX	56	—	—	—		137 8	136 32	135 55		4
	52	—	—	—		135 57	135 21	134 43		8
	48	—	—	—		134 47	134 11	133 32		12
	44	—	—	—		133 38	133 1	132 22		16
	40	—	—	—		132 30	131 52	131 13		20

In North Latitude

When Apparent Time is A.M. read the Azimuth from North to East.
When Apparent Time is P.M. read the Azimuth from North to West.

Weir's Azimuth Diagram. This diagram provides a graphical method of finding the azimuth. Figure 104 shows a skeleton diagram.

It is made up of two confocal series, one of hour-angle hyperbolas, the other of latitude ellipses. The mathematical reason for this is given in Volume III.

The azimuth is found from this diagram by :

(1) reversing the names of the latitude and declination, and when the latitude is 35°N., say, starting along the latitude ellipse from the point *K*, which is 35°S.

(2) marking the point *A* where this ellipse cuts the hyperbola corresponding to the given hour-angle.

(3) marking on the meridian the declination-point *B*, 20°S. if the declination is actually 20°N., and joining *AB*.

(4) drawing *OH* from the observer's position *O*, parallel to *BA* and cutting the horizon at *H*.

The angle *NOH* is the azimuth.

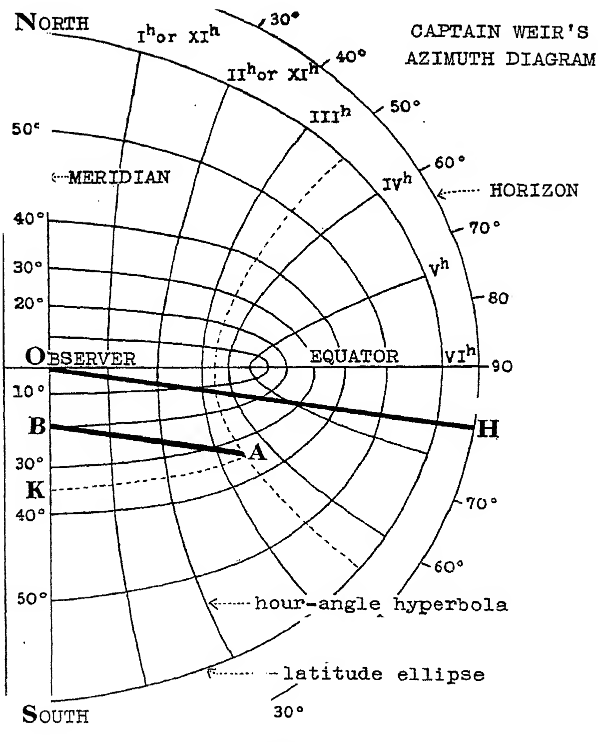


FIGURE 104.

The Latitude and Longitude of the Geographical Position. If it should happen that the altitude of the heavenly body is so large

that the position circle itself can be drawn, the geographical position of the heavenly body must be plotted.

Figure 105 shows that the latitude of the geographical position, *rx*, is equal to the declination of the heavenly body, *RX*.

The longitude of the geographical position is *gr*, which is the Greenwich hour angle of the heavenly body.

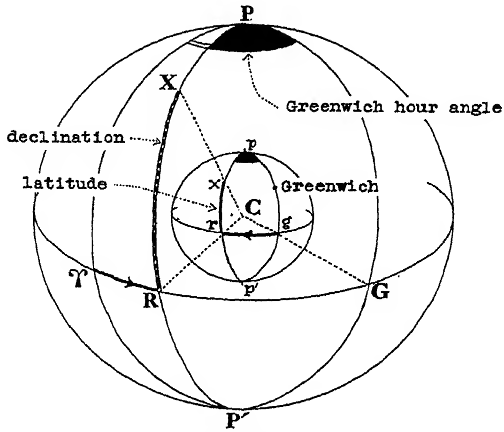


FIGURE 105.

Suppose, for example, that on the 3rd April 1937 at Z.T. 0925(—3) the deck watch showed $6^h25^m14^s$ when the Sun's altitude was taken, and that the deck watch was 43^s fast on G.M.T. Then:

Z.T.	0925 3rd April			
Zone	—3			
G.D.	0625 3rd April			
	h	m	s	
D.W.T.	6	25	14	
Error fast			43	
G.M.T.	6	24	31	3rd April
E	11	56	31	
G.H.A. Sun	18	21	02	≡275°15′·5W.

The longitude of the Sun's geographical position is therefore 84°44′·5E., and its latitude, from the *Nautical Almanac*, is 5°10′·2N.

If a star were observed—Regulus, say—at Z.T. 1900(+2) on

the 3rd April 1937, the deck watch showing $9^h04^m17^s$ and having an error of 13^s slow on G.M.T., the working would be :

Z.T.	1900 3rd April	
Zone	+2	
G.D.	2100 3rd April	

	h m s	
D.W.T.	9 04 17	
Error slow	13	
G.M.T.	21 04 30	3rd April
R	12 47 05	
R.A.M. (Greenwich)	9 51 35	
R.A. Regulus	10 05 04	
G.H.A. Regulus	23 46 31	≡356°37′·8W.

The longitude of the star's geographical position is therefore 3°22′·2E.

The declination of Regulus is 12°16′·3N., and the latitude of its geographical position is therefore the same.

THE LONGITUDE METHOD

In this method, the point where the position circle cuts a chosen parallel of latitude is found—figure 106—and the position line is drawn through this point at right-angles to the bearing.

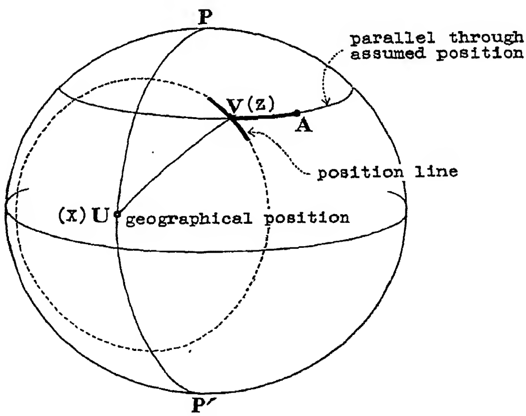


FIGURE 106.

Figure 107 shows the difference between the intercept and the longitude methods. In practice, however, when the Mercator chart is used, the point Z can be plotted directly because its longitude is found. On a plotting chart it would have to be plotted with reference to the D.R. as shown.

The steps by which the longitude of Z is found are :

(1) Measure the altitude of the heavenly body ; correct it and so find the true zenith distance.

(2) Using the latitude of the selected parallel to give the side PZ in the triangle PZX, and the declination to give PX, and the true zenith distance ZX, solve the triangle for the angle at the pole, which is the hour angle measured from the meridian through Z.

(3) To this hour angle, apply E or R and R.A., and so find L.M.T.

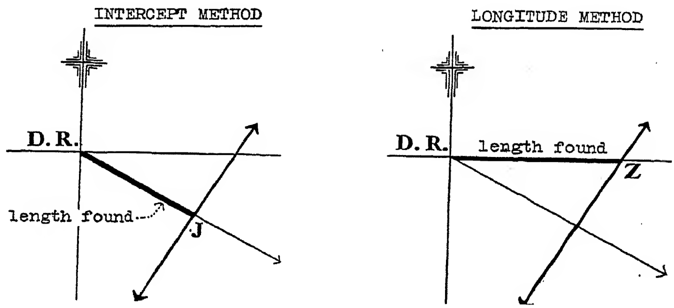


FIGURE 107.

The difference between this L.M.T. and the G.M.T. of the observation is the longitude of Z.

Consider, for example, the Sun sight given on page 136. The working of this sight by the longitude method would be :

Sext. Alt.	26°18′·0
True Alt.	26°24′·0
T.Z.D.	63°36′·0 (as before)
Sun's declination	5°13′·5N.
E	$11^h56^m33^s{\cdot}6$
Z.T.	0750 3rd April
Zone	+2
G.D.	0950 3rd April
	h m s
D.W.T.	9 51 33
Error slow	31
G.M.T.	9 52 04 3rd April

To find the angle ZPX:

PZ	55°29′·0	0·084	09
PX	84°46′·5	0·001	80
	29°17′·5		
ZX	63°36′·0		
+	92°53′·5	4·860	17
−	34°18′·5	4·469	74
∠*ZPX* =	4^h5^m	9·415	80

Since this is a 'morning' sight, the Sun bears east and the hour angle lies between 12^h and 24^h.

∴	H.A.T.S.	=	$24^h - 4^h5^m31^s$
		=	$19^h54^m29^s$
	E		11 56 34 (subtracted)
	L.M.T.		7 57 55
	G.M.T.		9 52 04
	Longitude of *Z*		1 54 09
		=	28°32′·2W.

(The longitude is *west* because the L.M.T. is less than the G.M.T. That is, G.M.T. is *best*.)

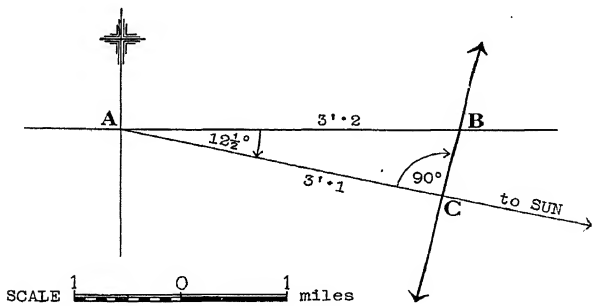


FIGURE 108.

By calculation, or from tables, the Sun's azimuth is seen to be N.102½°E. If, then, the point 34°31′N., 28°32′·2W. is marked on the Mercator chart and through it a straight line is drawn in the direction N.(102½°+90°)E., that line is the position line. (Figure 108.)

Agreement between Intercept and Longitude Methods. That this position line is the same as the one found by the intercept method can be seen from simple calculation.

Longitude A	28°36′·0W.
Longitude B	26°32′·2W.
d'long	3′·8
departure	3′·2

But AC is equal to AB cos $12\frac{1}{2}°$. Therefore :

$$\text{intercept} = 3'{\cdot}2 \times \cos 12\tfrac{1}{2}°$$
$$= 3'{\cdot}1$$

This is the length obtained by the other method.

CHAPTER XV

THE SEA AND AIR NAVIGATION TABLES *

In Chapter XIV it was explained that the intercept method of finding the position line is based on a comparison between the true zenith distance obtained from a sextant altitude and the calculated distance of the heavenly body's geographical position from an arbitrarily chosen position. The position chosen in the subsequent examples was the D.R., and there are certain advantages attached to this position when a number of position lines have to be plotted for ' simultaneous ' sights ; but it is not necessary to work from the D.R. and, when a single position line is to be found, there are definite advantages to be obtained by working from a position other than the D.R.

By working from this special position and solving the spherical triangle by dividing it into two right-angled triangles, it is possible, for reasons to be explained, to build compact and easily-entered tables giving the actual solutions of these triangles and thus avoiding the arithmetical labour of the cosine-haversine method. Tables that would solve the spherical triangle *PZX* for every possible combination of hour angle, latitude and declination without the necessity for interpolation cannot be constructed : space does not permit. But, when the triangle is divided, tables can be constructed quite easily for every degree of hour angle and latitude, and it is not difficult to interpolate for any factor involving the declination.

The Special Position. The special position from which the sight is worked is therefore one in which the latitude is taken to the nearest degree, and the longitude is chosen so that it combines with the Greenwich hour angle to give a local hour angle to the nearest degree in arc or four minutes in time.

(1) *The Latitude of the Special Position.* The choice of the latitude offers no difficulty. If the D.R. latitude is 47°15′N., the latitude of the position from which the sight must be worked is 47°N.

(2) *The Longitude of the Special Position.* This can be quickly found when an almanac tabulating G.H.A. in arc is used. If, for example, the D.R. longitude is 26°38′W. and the Greenwich hour angle is 53°18′·3, the longitude of the special position will be that

* These tables are published by and can be obtained from Henry Hughes & Sons, Ltd., 59 Fenchurch St., London, E.C.3.

nearest to 26°38′ which makes the local hour angle a whole degree. Thus :

G.H.A.	53°18′·3
Long. W.	26°18′·3
L.H.A.	27°00′·0

Had the D.R. longitude been 26°38′E., the L.H.A. would have been :

G.H.A.	53°18′·3
Long. E.	26°41′·7
L.H.A.	80°00′·0

The choice of 25°41′·7 for the longitude is undesirable because it gives a displacement of the D.R. position larger than necessary.

When an almanac tabulating the right ascension of the Mean Sun and the equation of time (either directly or as R and E) in units of time is used, the Greenwich hour angle should be found in units of time and then converted into arc.

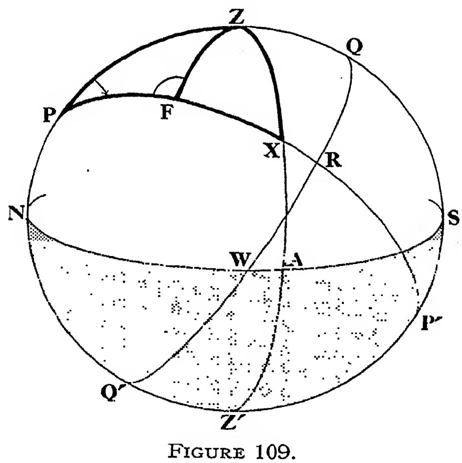


FIGURE 109.

If, on the other hand, it is intended to work in units of time throughout, the longitude should be adjusted in such a way that the local hour angle is a multiple of 4^m. This longitude, when converted back into units of arc, is the longitude of the special position.

The Division of the Spherical Triangle *PZX*. This can be effected by dropping the perpendicular either from Z on to PX, or from X on to PZ. In the method employed in the tables described in this chapter, the perpendicular, of length p, is dropped from Z as shown in figures 109 and 110.

F is the foot of this perpendicular, and the declination of F, which takes the same name as the observer's latitude, is K.

In these figures, X is shown with a northerly declination less than K. FX is therefore equal to $(K-d)$. But this northerly declination may be greater than K. If so, FX is equal to $(d-K)$. Also, when the declination is south, FX is clearly $(K+d)$. Hence, in general, FX is equal to $(K \sim d)$, and the two quantities are combined according to the ordinary rule for finding the difference of latitude between two places on the Earth: opposite names, add; same names, subtract.

PF is $(90°-K)$; PZ is $(90°-\text{latitude})$; and ZX is the zenith distance to be calculated. This, however, is $(90°-XA)$, which is 90° minus a quantity that may be termed the calculated altitude. In order to save the step in which the true altitude is subtracted from 90° to give the true zenith distance, the tables give this calculated altitude, and the intercept is found by comparing two altitudes, the true and the calculated. The rule for laying off an intercept found by comparing two zenith distances is therefore reversed.

The two right-angled triangles into which ZF divides the triangle PZX are known as the *time triangle* (PZF) and the *altitude triangle* (ZXF), and they are solved by Napier's rules, which are explained on page 251 of the Appendix.

Figure 110 shows the two triangles, drawn on the plane of the horizon, and the arrangement of their circular parts according to Napier.

To Solve the Time Triangle. In the time triangle PZ, the co-latitude, and h, the hour angle, are known, and it is required to find K and p so that the altitude triangle may be solved. Napier's rules give both quantities directly:

$$\sin(90°-h)=\tan(90°-K)\tan l$$

i.e. $$\cot K=\cos h \cot l \quad \ldots\ldots \quad (1)$$

and $$\sin p=\sin h \cos l \quad \ldots\ldots \quad (2)$$

It is therefore a simple matter to tabulate the values of K and p, or some convenient function of them, for every degree of latitude and hour angle, and in the actual tables the first column of Table I gives K itself in degrees and minutes of arc to one place of decimals. The second column, marked A, gives not p but, for a reason that will be apparent when the altitude triangle is solved, log sec p (multiplied by 10^5).

To Solve the Altitude Triangle. Once K is known, the declination can be applied and $(K \sim d)$ found. Napier's rules then give:

$$\sin(\text{alt.})=\cos(K \sim d)\cos p$$

i.e. $$\operatorname{cosec}(\text{alt.})=\sec(K \sim d)\sec p \quad \ldots\ldots \quad (3)$$

By using the secant and cosecant instead of the sine and cosine, the logarithmic work is simplified.

In logarithmic form, equation (3) is :

$$\log \operatorname{cosec} (\text{alt.}) = \log \sec (K \sim d) + \log \sec p$$

—and the reason for column A in Table I should now be clear.

Table II gives, in parallel columns headed B and C, logarithmic secants and cosecants multiplied by 10^5. Hence $\log \sec (K \sim d)$ is

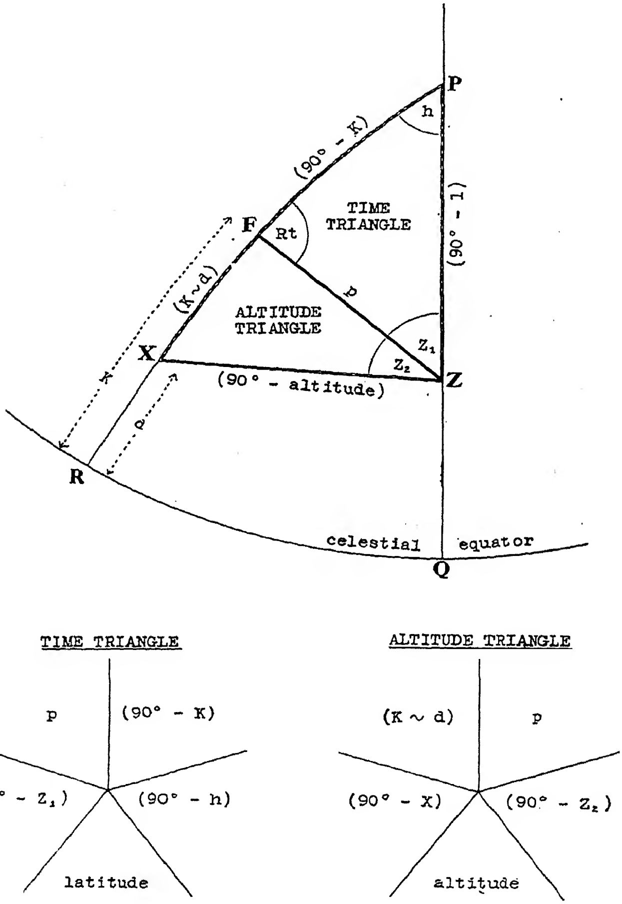


FIGURE 110.

found from column B, and by entering column C with the sum $(A+B)$, A being the value of $\log \sec p$ found from Table I, the altitude is read directly.

Suppose, for example, that the Greenwich hour angle of a heavenly body is 299°24′·5 and its true altitude 64°16′·5, the declination being

38°42′·7N., and that the D.R. position at the time of the observation is 21°17′N., 38°15′E. Then :

G.H.A.	299°24′·5
Long. E.	38°35′·5
L.H.A.	338°00′·0W. or 22°E.

Assumed latitude 21°N.

With latitude 21° and H.A. 22° enter Table I and take out K and A, and afterwards with ($K \sim d$) equal to 16°13′·3, enter Table II and take out B. Thus :

Dec.	38°42′·7N.		
K	22°29′·4N.	A	2833
$K \sim d$	16°13′·3	B	1764
		$A+B$	4597

Now enter column C of Table II with ($A+B$) equal to 4597, and take out the calculated altitude.

Calc. Alt.	64°06′·0
True Alt.	64°16′·5
Intercept	10′·5 towards

This intercept, it must be remembered, is laid off, not from the D.R. position, but from the special position 21°N., 38°35′·5E.

To Find the Azimuth. The dropping of a perpendicular from Z and the consequent division of the angle PZX, make it necessary to find the azimuth as the sum of the two angles PZF and FZX.

Napier's rules applied to the time triangle for the angle PZF and the altitude triangle for FZX give :

$$\sin l = \tan (90° - Z_1) \tan (90° - h)$$

i.e.
$$\tan Z_1 = \operatorname{cosec} l \cot h$$

and
$$\sin p = \tan (K \sim d) \tan (90° - Z_2)$$

i.e.
$$\tan Z_2 = \tan (K \sim d) \operatorname{cosec} p$$

The component Z_1 can thus be tabulated directly for every degree of latitude and hour angle in Table I, and this is done in a fourth column marked Z_1. Since the azimuth is not required to the same degree of accuracy as the altitude, the values of Z_1 are given to one decimal of a degree.

The third column of Table I gives log cosec p, and is headed D.

The other quantity required for finding Z_2, log tan ($K \sim d$), is given in Table III, which is simply a table of logarithmic tangents, in units of the third decimal, for every tenth of a degree. The quantity E, found with argument ($K \sim d$), is then added to D, and the body of the table is entered with ($D+E$) to give Z_2, which is read from the argument columns.

Rule for Combining Z_1 and Z_2. So far it has been assumed that the azimuth of the heavenly body is the sum of Z_1 and Z_2, but this assumption is not general. Figure 111 shows that, when the

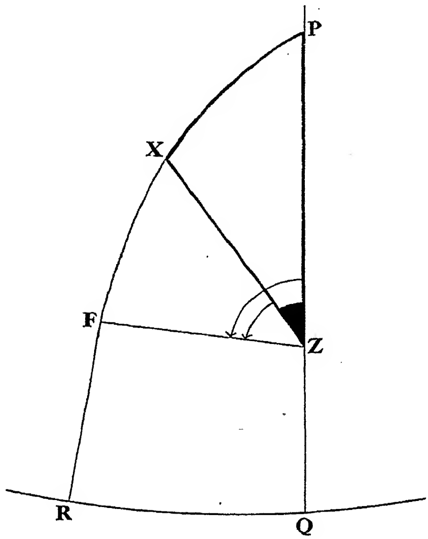


FIGURE 111.

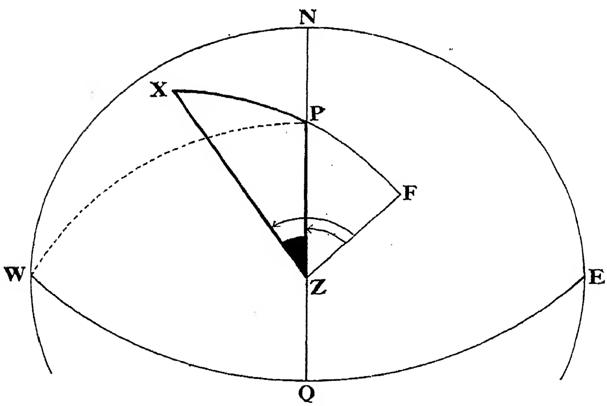


FIGURE 112.

declination is greater than K and of the same name, the azimuth is $(Z_1 - Z_2)$; and figure 112 shows that when the hour angle lies between 90° and 270° (that is, between 6^h and 18^h) the azimuth is $(Z_2 - Z_1)$

The azimuth is therefore ~Z_2) and the rule governing the sign may be stated thus :

The azimuth is the sum of Z_1 *and* Z_2 *except when the declination is greater than* K *and of the same name or when the local hour angle lies between* 90° *and* 270°. *The azimuth is then the difference between* Z_1 *and* Z_2, *the smaller angle being taken from the larger.*

Use of the Tables. In the actual use of the tables, the quantities giving the azimuth are taken out at the same time as the quantities giving the altitude. The relevant extracts from the tables show these quantities for the example already worked for the altitude, and it will be seen that the only interpolation which has to be done occurs in Table II for the decimal part of $(K \sim d)$. The quantities involved, however, are so small that this offers no difficulty.

TABLE I—LATITUDE 21°

h	K	A	D	Z_1		h	K	A	D	Z_1	
°	° ′			°	°	°	° ′			°	°
0	21 00·0	0·0	—	90·0	**180**	**45**	28 29·8	12428	180	70·3	**135**
1	21 00·2	5·8	1788	89·6	**179**	**46**	28 55·5	13021	173	69·6	**134**
2	21 00·7	23·1	1487	89·3	**178**	**47**	29 22·4	13631	166	69·0	**133**
...	—	—	—	—	...	...	—	—	—	—	...
22	22 29·4	2833	456	81·8	**158**	**67**	44 29·5	29127	66	49·8	**113**
23	22 38·2	3101	438	81·4	**157**	**68**	45 42·0	30039	63	48·4	**112**
...	—	—	—	—	...	...	—	—	—	—	...
44	28 05·2	11850	188	70·9	**136**	**89**	87 23·8	44522	30	2·8	**91**
45	28 29·8	12428	180	70·3	**135**	**90**	90 00·0	44567	30	0·0	**90**
	180°$-K$	A	D	$-Z_1$	h		180°$-K$	A	D	$-Z_1$	h

TABLE II

	16°		
	B	C	
′			′
00·0	1716	55966	**60·0**
00·5	1718	55944	**59·5**
01·0	1719	55922	**59·0**
...	—	—	...
13·0	1763	55398	**47·0**
13·5	1765	55376	**46·5**
...	—	—	...
29·5	1824	54687	**30·5**
30·0	1826	54666	**30·0**
	B	C	
	163°		

	64°		
	B	C	
′			′
00·0	35816	4634	**60·0**
00·5	35829	4631	**59·5**
01·0	35842	4628	**59·0**
...	—	—	...
06·0	35972	4597	**54·0**
06·5	35985	4594	**53·5**
...	—	—	...
29·5	36588	4454	**30·5**
30·0	36602	4451	**30·0**
	B	C	
	115°		

TABLE III

Enter for E with argument $(K\sim d)$
Enter for Z_2 with argument $(D+E)$

$K\sim d$ or Z_2	0′	6′	12′	18′	...	42′	48′	54′	60′
	·0	·1	·2	·3	...	·7	·8	·9	1·0
°									
0	—	7242	7543	7719	—	8087	8145	8196	8242
1	8242	8283	8321	8356	—	8472	8497	8521	8543
...	—	—	—	—	—	—	—	—	—
16	9457	9460	9463	9466	—	9477	9480	9483	9485
...	—	—	—	—	—	—	—	—	—
39	9908	9910	9911	9913	—	9919	9921	9922	9924
...	—	—	—	—	—	—	—	—	—
89	1758	1804	1855	1913	—	2281	2457	2758	—

The example, set out in full, is :

G.H.A.	299°24′·5
Long. E.	38°35′·5
L.H.A.	338°00′·0W. or 22°E.

Assumed latitude 21°N.

Dec.	38°42′·7N.						
K	22°29′·4N.	A	2833	D	456	Z_1	+81°·8
$K\sim d$	16°13′·3	B	1764	E	9464		
		$A+B$	4597	$D+E$	9920	Z_2	−39°·8

Calc. Alt.	64°06′·0	Az.	N.42°·0E.
True Alt.	64°16′·5		
Intercept	10′·5 towards		

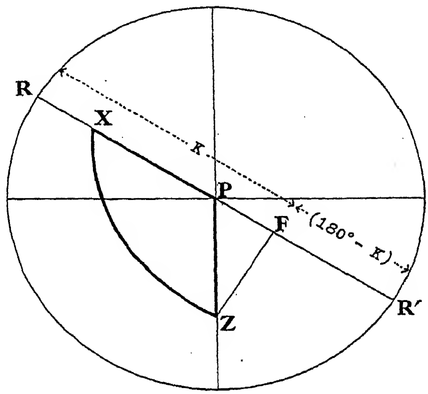
The Value of $(K\sim d)$. Since $(K\sim d)$ is a combination of the declination of the heavenly body and the declination of the foot of the perpendicular from Z on to the heavenly body's meridian, it can never be greater than 90° when the heavenly body is above the horizon ; but when the hour angle lies between 90° and 270°, K by itself is greater than 90°, as figure 113 shows.

In these circumstances Table I is entered from the bottom for an angle that is the supplement of the one marked on the left. But the quantity obtained from the tables is less than 90° and is equal to the length FR'. For this reason the first column marked K at the top is marked $(180°-K)$ at the bottom, and for hour angles

between 90° and 270° the quantity obtained from it must be subtracted from 180° in order to give K.

NOTE. Although the foregoing theory is based on a westerly hour angle that increases from 0° to 360°, so that the value of K is considered for values of h between 90° and 270°, the tables themselves do not go beyond 180°. When, therefore, h is greater than 180°, it must be subtracted from 360° and the result regarded as an easterly hour angle, so that the azimuth must be named east.

Summary of Procedure. Although these tables can be used for an hour angle expressed in units of time, the longitude of the special position can be obtained more quickly and with less likelihood of



PLANE OF THE CELESTIAL EQUATOR

FIGURE 113.

error if the hour angle is expressed in arc. For this reason the procedure recommended is, in summary:

(1) Find the Greenwich hour angle in arc.

(2) Choose that longitude nearest the D.R. longitude which will make the local hour angle an integral number of degrees.

(3) Choose the integral degree of latitude nearest to the D.R. latitude.

(4) With this chosen latitude and the local hour angle, enter Table I and take out K, A, and D and $+Z_1$. If the local hour angle lies between 90° and 180°, enter the table from the bottom and take out $(180°-K)$, A, D and $-Z_1$.

(5) Find $(K{\sim}d)$ according to the ordinary rule for finding the difference of latitude between two places.

(6) Enter column B of Table II with the argument $(K{\sim}d)$ and take out B.

(7) Enter column C of Table II with argument $(A+B)$ and read the calculated altitude from the angle column at the side.

(8) Enter Table III with argument $(K{\sim}d)$ and take out E.

(9) Enter Table III with argument $(D+E)$ and read Z_2.

(10) Combine Z_1 and Z_2 to give the azimuth and name it from the elevated pole, east or west according as the local hour angle is

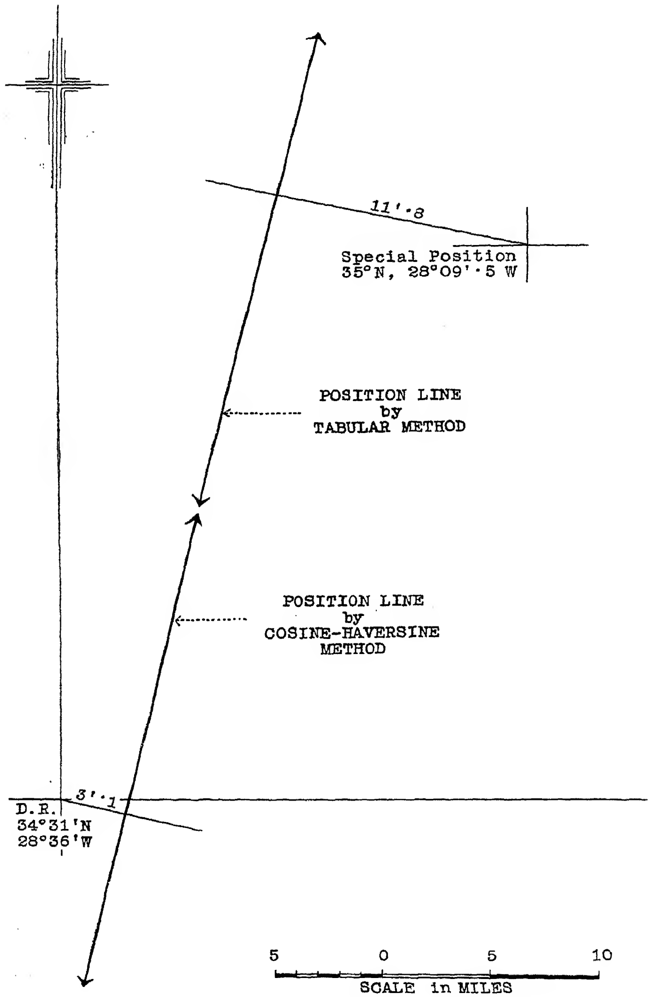


FIGURE 114.

greater or less than 180°. Z_1 and Z_2 are added except when the declination is greater than K and of the same name, or when the local hour angle lies between 90° and 270°. The smaller quantity is then taken from the larger.

(11) Find the intercept by taking the difference between the true altitude and the altitude found from the tables. If the true altitude is the smaller, the intercept is *away*. If the true altitude is the larger, the intercept is *towards*.

(12) Lay off the intercept from the point chosen in (2) and (3).

In order to compare this method with the cosine-haversine method, the sights worked from the D.R. position on pages 136 and 138 of Chapter XIV are here worked from the special positions that enable the S.A.N. Tables to be used, and it will be seen that the same position lines are obtained. It will also be seen that this tabular method enjoys a considerable advantage in the saving of time and space. The tables cover all latitudes and are therefore suitable for polar navigation by aircraft.

At Z.T. 0750(+2), *3rd April* 1937, *the D.R. was* 34°31′*N.*, 28°36′*W.*, *and the deck watch showed* 9ʰ51ᵐ33ˢ *when the observed altitude of the Sun was* 26°18′·0. *The deck watch was* 31ˢ *slow on G.M.T.: the index error was* —2′·6, *and the height of eye* 32 *feet. Draw the position line.*

		Sun's Declination
Z.T.	0750 3rd April	5°11′·7N.
Zone	+2	+1′·8
G.D.	0950 3rd April	5°13′·5N.

	h m s	E h m s
D.W.T.	9 51 33	11 56 32·2
Error slow	31	+1·4
G.M.T.	9 52 04 3rd April	11 56 33·6
E.	11 56 34	

G.H.A.	21 48 38	Sext. Alt.	26°18′·0
	= 327°09′·5	I.E.	—2′·6
Long. W.	28°09′·5	Obs. Alt.	26°15′·4
H.A.T.S.	299°W. or 61°E.	Corrⁿ.	+8′·6
		True Alt.	26°24′·0

Assumed latitude 35°N.

Dec.	5°13′·5N.					
K	55°18′·1N.	A	15637	D	145	Z_1 +44°·0
$K \sim d$	50°04′·6	B	19263	E	78	
		$A+B$	34900	$D+E$	223	Z_2 +59°·1
						Az. N.103°·1E.

Calc. Alt.	26°35′·8
True Alt.	26°24′·0
Intercept	11′·8 away

In figure 114 the position lines obtained with this intercept, laid off from 35°N., 28°09′·5W., and the intercept of 3′·1 found by the cosine-haversine method and laid off from the D.R., 34°31′N., 28°36′W., are shown on the same plotting chart.

At Z.T. 2110(—8), 3*rd April* 1937, *the D.R. was* 21°15′*N.*, 115°38′*E.*, *and the deck watch showed* $1^h10^m43^s$ *when the sextant altitude of Arcturus was* 26°25′·2. *The deck watch was* 15^s *slow on G.M.T.; the index error was* +2′·3*; and the height of eye* 28 *feet. Draw the position line.*

		R (h m s)
Z.T.	2110	12 45 35·7
Zone	—8	11·7
G.D.	1310 3rd April	12 45 47·4

	h m s		
D.W.T.	1 10 43	*Arcturus*	
Error slow	15	R.A.	$14^h12^m50^s$
		Dec.	19°30′·3N.
G.M.T.	13 10 58 3rd April		
R	12 45 47		
R.A.M.(G.)	25 56 45		
R.A.✳	14 12 50	Sext. Alt.	26°25′·2
		I.E.	+2′·3
G.H.A.✳	11 43 55		
	= 175°58′·8	Obs. Alt.	26°27′·5
Long. E.	116°01′·2	Corrⁿ.	—7′·2
H.A.✳	292°W. or 68°E.	True Alt.	26°20′·3

Assumed latitude 21°N.

Dec.	19°30′·3N.						
K	45°42′·0N.	*A*	30039	*D*	63	Z_1	+48°·4
K∼*d*	26°11′·7	*B*	4706	*E*	9692		
		A+*B*	34745	*D*+*E*	9755	Z_2	+29°·6
						Az.	N.78°·0E.

Calc. Alt.	26°42′·0
True Alt.	26°20′·3
Intercept	21′·7 away

Figure 115 shows the position line obtained by plotting from 21°N., 116°01′·2E. with an intercept of 21′·7 away, and the position

line obtained by plotting from 21°15′N., 115°38′E. with an intercept of 3′·6 away, and it is seen that in this example, as in the other,

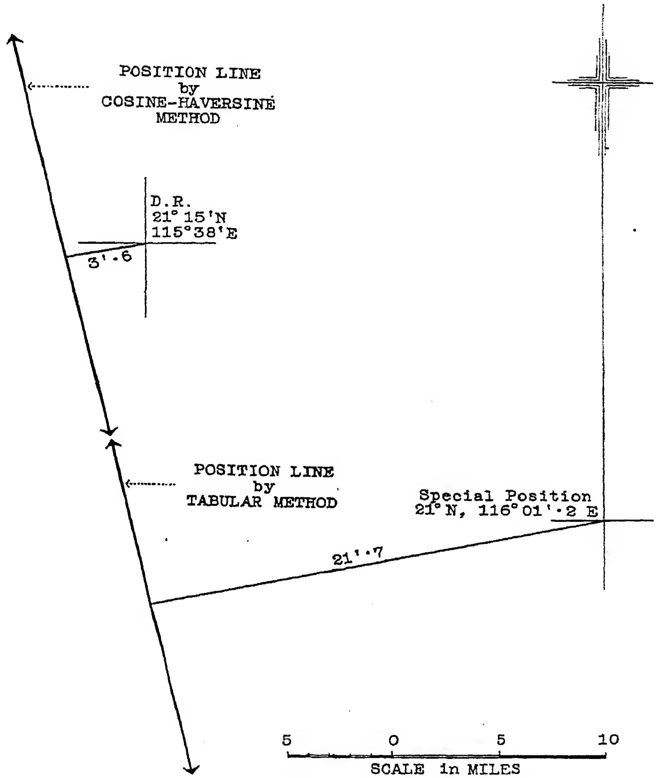


FIGURE 115.

the two position lines coincide although there is a considerable difference between the D.R. position and the special position in both examples.

CHAPTER XVI

ALTITUDE-AZIMUTH TABLES

If tables could be constructed so as to give the solutions of the spherical triangle for every possible combination of latitude, declination and hour angle without interpolation, these tables would be ideal. They would give the navigator at a single glance the quantities that he now has to find by some logarithmic or tabular method. But, as stated in the last chapter, the construction of such comprehensive tables is not possible because of the space they would occupy, and when tables are constructed for a limited number of combinations, as for every degree of latitude, declination and hour angle, interpolation between the quantities at once becomes necessary, increasing in awkwardness as the tables themselves decrease in bulk. The problem, therefore, is to find a convenient compromise between bulk and interpolation.

Arrangement of Altitude-Azimuth Tables. This is done by dividing the tables into a number of volumes, each covering a convenient belt of latitude, and making each volume sufficiently large to contain solutions of the spherical triangle for every degree of hour angle and latitude, and all necessary declinations, the solutions for same and opposite names being tabled on opposite pages as a rule. The United States Navy Department publication, H.O.214, covers a 10°-belt of latitude and tabulates for every half-degree of declination, except in high latitudes. The tables used by the Royal Air Force cover 5°-belts of latitude and contain separate tabulation for the stars normally used in navigation, this tabulation being based on the exact declinations of the stars.

Method of Interpolation. In order to facilitate interpolation, two quantities are given in addition to the altitude and azimuth corresponding to a particular combination of the main arguments, these quantities being proportional to the changes in altitude that result from changes of 1′ in the declination and the hour angle. The actual lay-out varies with different publications. Declination and hour angle may be read vertically and horizontally, for example, instead of horizontally and vertically as they are in the extract given here, which is from H.O.214. But the principle remains unaltered, whatever the arrangement, and this extract may be taken as typical of the altitude-azimuth tables that can be used for working sights.

DECLINATION *SAME* NAME AS LATITUDE

LAT. 35°	H.A.	5°00′				5°30′			
		Alt.			Az.	Alt.			Az.
			Δd	Δh			Δd	Δh	
	00°	60°00′·0	1·0	01	180°·0	60°30′·0	1·0	01	180°·0
	1	—	—	—	—	—	—	—	—
	2	—	—	—	—	—	—	—	—
	...								
	60	27°15′·6	60	80	104°·0	27°33′·6	60	80	103°·6
	61	26°27′·7	60	80	103°·3	26°45′·7	60	80	102°·9
	...	—	—	—	—	—	—	—	—

These small quantities, 60 and 80, are the actual changes in altitude for one minute of arc multiplied by 100. If, then, the altitude is required for an hour angle of 60°20′·5 and a declination of 5°12′·3N. in latitude 35°N., the proportions to be applied to the altitude taken out for an hour angle of 60° and a declination of 5° are :

For a 12′·3 *difference in declination* $\frac{12'\cdot3\times60}{100} = 7'\cdot4$

For a 20′·5 *difference in hour angle* $\frac{20'\cdot5\times80}{100} =16'\cdot4$

This multiplication is effected by a special table, provided inside the back cover of the volume.

It will also be noticed from the neighbouring quantities that, when the declination is increased from 5°00′ to 5°30′, the altitude is increased from 27°15′·6 to 27°33′·6 ; and that when the hour angle is increased from 60° to 61°, the altitude is decreased from 27°15′·6 to 26°27′·7. The sign of the 12′·3 increase in declination is therefore plus, and that of the 20′·5 increase in hour angle is minus. The required altitude is thus :

Altitude from Tables	27°15′·6
Correction for declination	+ 7′·4
Correction for hour angle	−16′·4
Calculated altitude	27°06′·6

The azimuth in the given circumstances is taken as N.104°W. Greater accuracy is not required in ordinary sight-work, but if it is required at any time, it can be obtained by interpolation.

In the example just given, the quantities were obtained by working *forward* from the nearest tabular entries which happen to be lower than the given values of the hour angle and declination. If the nearest tabular entries happen to be higher than these values, it is necessary to work *backward* from them. This procedure will give greater accuracy, particularly in the interpolation for the hour

angle, than the procedure that limits itself to working forward from the nearest *lower* tabular entries.

Suppose, for example, that the hour angle is 60°39′·5 and the declination 5°16′·5N., the latitude being 35°N. The interpolation factor for the hour angle is 20′·5 and that for the declination is 13′·5, the nearest tabular entries being an hour angle of 61° and a declination of 5°30′.

The proportions to be applied are :

For a 13′·5 *difference in declination* $\frac{13'\cdot5\times60}{100}=8'\cdot1$

For a 20′·5 *difference in hour angle* $\frac{20'\cdot5\times80}{100}=16'\cdot4$

But when the declination is decreased from 5°30′ to 5°00′, the altitude is decreased ; and when the hour angle is decreased from 61° to 60°, the altitude is increased. The signs of the quantities are therefore the reverse of the signs in the previous example, where the work was carried forward from the tabular entries.

The required altitude is thus :

Altitude from Tables	26°45′·7
Correction for declination	− 8′·1
Correction for hour angle	+16′·4
Calculated altitude	26°54′·0

Sights Worked from Altitude-Azimuth Tables. Sights can be worked from these tables by two methods. A special position can be chosen as one is chosen when the S.A.N. Tables are used. The hour angle is then a whole number and the only interpolation is for declination. The choice of such a position therefore lessens the time taken in working the sight and tends to increase accuracy. Otherwise the position from which the sight is worked must be taken to the nearest degree of latitude and interpolation carried out for both declination and hour angle. This procedure has the advantage of simple plotting if a number of ' simultaneous ' sights have to be worked, because the longitude employed throughout is the D.R. longitude.

Worked from this special position with the aid of these altitude-azimuth tables, the sight already worked by the cosine-haversine method on page 136 and by the tabular method on page 157 reads thus :

D.R. position $\begin{cases}34°31'\text{N.} \\ 28°36'\text{W.}\end{cases}$ Sun's declination 5°13′·5N. / True altitude 26°24′·0

G.H.A. 327°09′·5 (as calculated on page 157)
Long. W. 28°09′·5

H.A.T.S. 299°W. or 61°E.

Assumed latitude 35°N.

The tables are entered for a declination of 5°N. (difference, 13′·5) and hour angle 61°, and (see extract) the quantities taken out are :

Altitude	26°27′·7	△d 60	Az. N.103°E.
Corrⁿ. △d	+8′·1		
Calc. Alt.	26°35′·8		
True Alt.	26°24′·0		
Intercept	11′·8 away		

This intercept, it will be noted, is the same as that obtained by the tabular method, and since the sight is worked from the same chosen position, the position line obtained must be the same.

If the sight is worked from a position, the latitude of which is taken to the nearest degree, the longitude being that of the D.R., the hour angle does not work out to a whole degree and the sight reads :

D.R. position	34°31′N.	Sun's declination	5°13′·5N.
	28°36′W.	True altitude	26°24′·0

G.H.A.	327°09′·5
Long. W.	28°36′·0
H.A.T.S.	298°33′·5
=	61°26′·5E.

Assumed latitude 35°N.

As before, the tables are entered for a declination of 5°N. (difference 13′·5) and hour angle 61°, but the difference between this hour angle and the one with which the sight is being worked is 26′·5. The quantities taken out and the subsequent corrections are therefore :

Altitude	26°27′·7	△d 60	△h 80	Az. N.103°E.
Corrⁿ. △d	+ 8′·1			
Corrⁿ. △h	−21′·2			
Calc. Alt.	26°14′·6			
True Alt.	26°24′·0			
Intercept	9′·4 towards			

Figure 116 shows the position line obtained with this intercept laid off from the position 35°00′N., 28°36′W., and it is seen that the position line coincides with that obtained when the interpolation is restricted to the declination and the sight worked from the position 35°00′N., 28°09′·5W.

Accuracy of Altitude-Azimuth Tables. These examples should make it clear that altitude-azimuth tables of the pattern described afford the quickest and simplest means of working a sight. There is, however, the question of their accuracy.

When the altitude is found from a position that involves two interpolated quantities—both declination and hour angle, that is—a probable error of 0′·1 or 0′·2 is to be expected, but only if the altitude itself is normal and the interpolation is made from the *nearest* tabular entry in either direction. When the interpolation is restricted to one quantity, the declination, the probable error should not exceed 0′·1. The maximum error may be taken as about three times the probable error, but this is seldom encountered.

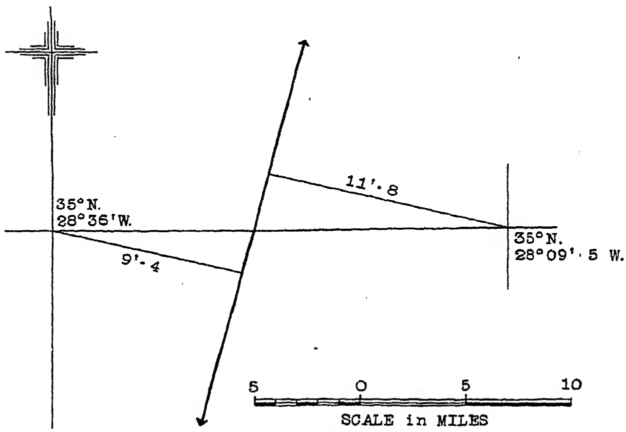


FIGURE 116.

The source of these errors lies in :

(1) the building-up error of three rounded-off quantities.

(2) the neglect of second differences.

(3) the lack of accuracy in the printed variations. The quantity 98, for example, represents any variation from 58′·5 to 59′·1.

Of these contributory factors, (1) is constant, but (2) and (3) increase with the size of the interpolating factor, and are four times and twice as large respectively with a factor of 59′·9 as with one of 29′·9.

In tables where the declination is given for every degree instead of every half-degree, the probable error is increased, but not sufficiently to render them unsuitable for surface navigation. When, however, the altitude itself is tabulated to the nearest minute instead of to one decimal place, the tables must be used with the same caution as that which is necessary when the *Air Almanac* is used in similar circumstances.

Summary of Procedure. In summary, the steps which have been elaborated in the foregoing examples may be stated thus :

Sight Worked from a Special Position involving One Interpolation.

(1) Find the Greenwich hour angle in arc and choose that longitude nearest the D.R. longitude which will make the local hour angle a whole number of degrees.

(2) Choose the latitude nearest in whole degrees to the D.R. latitude.

(3) Note the declination of the heavenly body and enter the tables with a declination which is the nearest whole or half-degree, and with the hour angle and the latitude.

(4) Take out the tabulated altitude and azimuth and the small quantity proportional to the change in altitude that results from a change of 1′ in declination.

(5) With this small quantity and the difference between the two declinations, enter the multiplication table and take out the correction to be applied to the tabulated altitude.

(6) Apply this correction with the sign indicated by the neighbouring tabulations in the main tables.

(7) Compare this calculated altitude with the true altitude, and name the azimuth from the elevated pole, east or west according as the hour angle is east or west.

Sight Worked from a Position involving Two Interpolations.

(1) Find the local hour angle on the D.R. meridian.

(2) As before.

(3) As before.

(4) Take out the tabulated altitude and azimuth, and the small quantities proportional to the changes in altitude that result from changes of 1′ in declination and hour angle.

(5) With these small quantities and the differences of declination and hour angle, enter the multiplication table and take out the corrections to be applied to the tabulated altitude.

(6) Apply these corrections with the sign indicated by the neighbouring tabulations in the main tables.

(7) As before.

CHAPTER XVII

MERIDIAN PASSAGE

In Chapter IX it was seen that when the hour angle of a heavenly body is 0^h or 12^h, the heavenly body is either due north or due south of the observer. Clearly a heavenly body situated thus has a particular importance because the position line obtained from it,

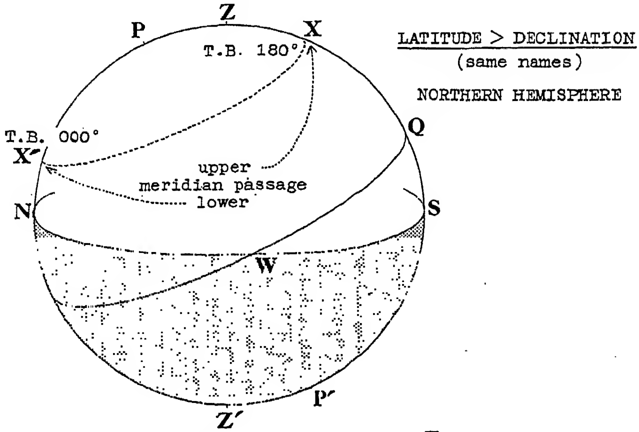


FIGURE 117a.

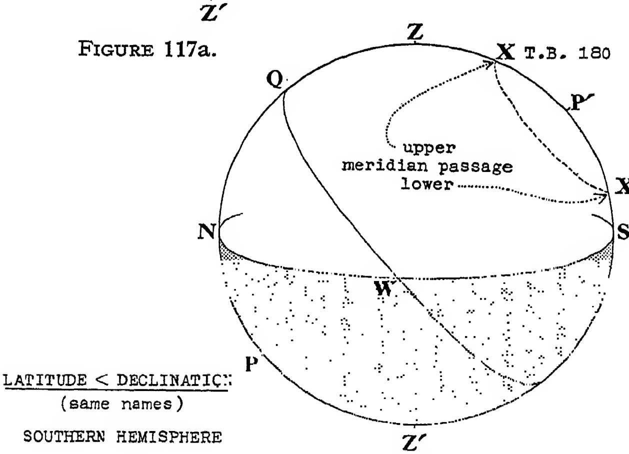


FIGURE 117b.

being at right-angles to the bearing, lies along the parallel of latitude and gives the observer's latitude exactly.

Upper Meridian Passage. This occurs when the body is on the observer's meridian, *PZP′* in figures 117a and 117b. The hour angle of the body is then 0^h.

In the northern hemisphere, the true bearing is 180° (figure 117a) or 000° according as the latitude is greater or less than the declination, but of the same name. When the latitude and declination have opposite names, the bearing is 180°. (Figure 118a.)

In the southern hemisphere these bearings are reversed. (Figures 117b and 118b.)

Lower Meridian Passage. This occurs when the body is on the meridian (*PZ'P'*) that differs in hour angle from the observer's meridian by 180° or 12^h, and it is sometimes referred to as the *meridian passage below the pole.* The hour angle of the body is then 12^h and in the northern hemisphere, when the body is visible, the bearing is always 000°, no matter which is the larger, the declination or the latitude. (Figure 117a.) In the southern hemisphere it is always 180°. (Figure 117b.)

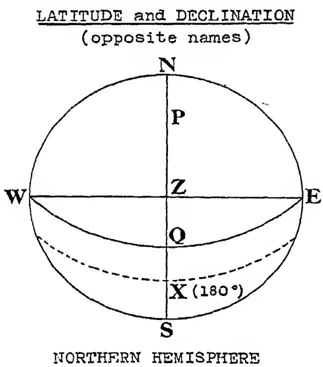


FIGURE 118a.

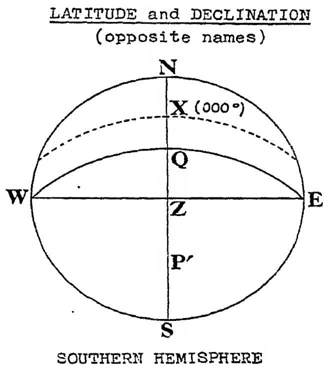


FIGURE 118b.

When the latitude and declination have opposite names, lower meridian passage can never be observed because the body is below the horizon.

Except in high latitudes for part of the year, the Sun's lower meridian passage cannot be observed. Also the number of stars visible at lower meridian passage is small. For these reasons the navigator engaged in sight-taking is concerned chiefly with the upper passage, and unless otherwise stated, the term *meridian passage* always refers to the *upper transit.*

Maximum Altitude. In figures 117a and 117b, and in figure 118a, *SX* is the altitude of the heavenly body, and this is clearly the greatest altitude the body can have when measured by a stationary observer whose zenith is *Z*. To a stationary observer the meridian altitude is thus the maximum altitude, and in order to obtain it he has only to watch the heavenly body through the sextant telescope and note the altitude the moment the heavenly body starts to dip. This, however, is not easily done at any time, and the method would

lead to appreciable error if it were employed by an observer in a ship moving in a direction other than east or west, because, to an observer moving thus, the Sun does not reach its greatest altitude at meridian passage.

Three factors govern the change in a heavenly body's altitude :

(1) the rotation of the Earth.
(2) the declination.
(3) the north–south component of the ship's movement.

The first ensures that, to an observer on the Earth, the body rises in the east, attains a maximum altitude, and then sets in the west.

The second does not, in practice, concern the navigator because any change in declination during the short period between meridian passage and the moment of greatest altitude is too small to matter. Nevertheless, any change in declination means a change in the position of X relative to Z, and that is a change in altitude.

The third introduces a small but significant complication because it is equivalent to a movement of Z. The Earth's rotation by itself would give the altitude its greatest value when the body reaches the meridian. But if the ship is moving towards the body, the altitude will increase for a further period until the rate at which the body is decreasing in altitude, due to the Earth's rotation, becomes equal to the rate at which the movement of the ship is increasing the altitude. The greatest altitude therefore occurs *after* meridian transit.

If the ship is moving away from the heavenly body, the greatest altitude occurs *before* meridian transit.

The time-difference between these two altitudes may lead to an error of 5′ if the ship's speed is high and her course approximately north or south. Only when the ship is steaming east or west will the greatest altitude occur at meridian passage.

Time of Meridian Passage. It is thus evident that a moving observer cannot, as a rule, take a meridian altitude by watching the heavenly body and noting when it starts to dip. Instead, he must work out the time when the heavenly body will be on his meridian and take the altitude at that moment. This he can do by simple approximation because the altitude changes slowly when the heavenly body is near the meridian, and an error of a few seconds in the calculated time of meridian passage makes no appreciable difference to the altitude. It is therefore customary to work out the time of meridian passage to the nearest minute.

The Sun's Meridian Passage. When the Sun is on the meridian, its hour angle is 0^h, and the approximate zone time of meridian passage can be found by taking this known hour angle and applying to it an approximate value of E, the longitude of the D.R. position and the zone description. The ship's zone time of meridian passage can now be calculated, and the time of meridian passage worked out for the new longitude which is obtained.

At Z.T. 0840(+1) on the 3rd April 1937, a ship was in a position 50°18′N., 19°33′W., steaming 320° at 20 knots. When was the Sun on her meridian?

First Approximation

	h m s	
H.A.T.S.	0 0 0	
E	11 56 35	(for 1200, 3rd April)
L.M.T.	12 03 25	
D.R. long. W.	1 18 12	
G.M.T.	13 21 37	3rd April
Zone (+1)	−1	
Approx. Z.T.	1222(+1)	3rd April

The approximate run to meridian passage is therefore ($12^h22^m - 8^h40^m$) at 20 knots; that is, 3^h42^m at 20 knots, or 74 miles.

By traverse table for 74′ at N.40°W.:

d'lat=56′·7N.	mean lat.=50°46′
dep. =47′·6W.	d'long = 1°15′·3W.

The approximate longitude at meridian passage is therefore 20°48′W.

The time of meridian passage can now be calculated with this longitude. Thus:

Second Approximation

	h m s	
H.A.T.S.	0 0 0	
E	11 56 36	(for 13^h22^m, G.M.T.)
L.M.T.	12 03 24	
Long. W.	1 23 12	(from new D.R.)
G.M.T.	13 26 36	3rd April
Zone (+1)	−1	
Z.T.	1227(+1)	3rd April

The first approximation gives the Z.T. of meridian passage when the ship is stationary. The second approximation shows that for practical sight-taking in the circumstances stated in the question, the Sun is on the meridian at 1227(+1).

This second approximation can be arrived at quickly by converting the difference of longitude (1°15′·3) into time (5^m01^s) and applying it directly to the zone time 1222(+1). The change is westerly—in the direction of the Sun's apparent motion, that is—and the 5^m must therefore be added. As before, the approximate Z.T. of meridian passage is 1227(+1). This method neglects any change in E, but no error is introduced because this change is too small to matter. If, however, the method is used for finding the meridian

passage of the Moon or a planet, disregard of the change in R.A. may give rise to an error which should not be ignored.

If extreme accuracy is required, a third approximation can be made, the run being taken for an interval of ($12^h27^m—8^h40^m$) or 3^h47^m instead of 3^h42^m. The change of longitude in this five-minute difference is, however, of no importance to the navigator in so far as it affects the calculated time of meridian passage. In practice he would avoid even a second approximation by taking his D.R. noon position from the chart and working from that to the nearest minute or half minute.

In the example just given, the 1200(+1) position is 51°09′N., 20°41′W., and the time of meridian passage found by using this longitude and working to the nearest minute is :

	h m	
H.A.T.S.	0 0	
E	11 57	(for 13^h00^m, G.M.T.)
L.M.T.	12 03	
Long. W.	1 23	(from 1200(+1) D.R.)
G.M.T.	13 26	
Zone (+1)	—1	
Z.T.	1226(+1) 3rd April	

The apparent discrepancy of one minute between the times found by the two methods arises partly because the times are taken to the nearest minute, $12^h26^m36^s$ being read as 1237, and $12^h26^m08^s$ as 1226. The actual difference is therefore just under half a minute, and the difference of longitude between the positions at 1200(+1) and 1222(+1) accounts for this. But a difference at noon of even one minute is of no practical significance because the Sun's change in altitude during that time is negligible.

A Star's Meridian Passage. When a star is on the meridian, its hour angle is 0^h, and the formula :

$$\text{H.A.}\ast + \text{R.A.}\ast = \text{L.M.T.} + \text{R}$$

—therefore reduces to :

$$\text{R.A.}\ast = \text{L.M.T.} + \text{R}$$

i.e.

$$\text{L.M.T.} = \text{R.A.}\ast - \text{R}$$

Of these components, the star's right ascension may be taken as constant, and a value of R can be found exactly as a value of E was found for the calculation of the Sun's time of meridian passage. This done, the working is the same as that for the Sun.

But the meridian passage of a star is not important, like that of the Sun, and in practice, the navigator confines himself to stars of convenient bearing and altitude which he may observe at morning and evening twilight when the horizon is good. He will not, as a rule, bother about the meridian passage of a star unless it is clear to him that the star will cross his meridian during one of these periods.

He will then work out its time of transit from the D.R. position that he has already decided upon for his star sights, and there will be no need for a second approximation. Thus :

At Z.T. 1830(—10) *during evening twilight on the 3rd April* 1937, *the D.R. position of a ship was* 40°20′*N.*, 154°05′*E. At what time did Procyon cross her meridian ?*

Z.T.	1830 3rd April
Zone	—10
G.D.	0830 3rd April

	h m s	
R.A. Procyon	7 36 02	
R	12 45 01	(for 8^h30^m, G.M.T.)
L.M.T.	18 51 01	
Long. E.	10 16 20	
G.M.T.	8 34 41	3rd April
Zone (—10)	+10	
Z.T.	1835(—10)	3rd April

A Star's Lower Meridian Passage. When a star crosses the meridian below the pole, its hour angle is 12^h, and the formula therefore becomes :

Apart from this adjustment, the steps by which the time of the lower transit is found do not differ from those giving the time of the upper transit. For example :

At what time on the 3rd April 1937 *will Canopus cross the meridian below the pole to an observer in* 51°42′*S.*, 57°51′*W.* ?

	h m s	
R.A. Canopus	6 22 33	
	12	
	18 22 33	
R	12 45 36	(for 12^h00^m, G.M.T.)
L.M.T.	5 36 57	
Long. W.	3 51 24	
G.M.T.	9 28 21	3rd April
Zone (+4)	—4	
Z.T.	0528(+4)	3rd April

In this example R has been taken from the *Nautical Almanac* for a G.M.T. of 12^h because there is nothing in the question to suggest the rough time of transit. A more accurate value of R would therefore be obtained for a G.M.T. of 9^h28^m. This value is $12^h45^m11^s$, and the zone time of transit becomes 0529(+4).

A Planet's Meridian Passage. The time of this can be found exactly as the time of a star's transit is found. For convenience, however, the L.M.T. of meridian passage at Greenwich is given in the *Nautical Almanac* for each of the four navigational planets, and since the daily difference in the times of transit is small, this time may also be taken as the approximate L.M.T. of passage across the observer's meridian, whatever its longitude.

The daily difference in the times of a planet's transit may, however, be as much as 4^m. In these circumstances, therefore, if an accurate time of transit across a particular meridian is wanted, it must be found either by the star method, in which the planet's right ascension at meridian passage is itself found with sufficient accuracy by successive approximation, or by the method described in the next section, which applies a correction to the given time of transit.

The Moon's Meridian Passage. This is important in tidal prediction. The times of upper and lower transits at Greenwich are given for each day throughout the year in the *Nautical Almanac*, and it is convenient to work from this information when the time of transit across another meridian has to be found, rather than to use the ordinary method that suffices for a star. But the change in the Moon's right ascension is so rapid that it must always be taken into account. For this reason the L.M.T. of the Moon's meridian passage at Greenwich can be taken as only a rough approximation of the L.M.T. of the transit across some other meridian.

It was shown on page 118 of Chapter XI that the lunar day is longer than a mean solar day by an average of 50^m. This means that the Moon crosses an observer's meridian later each day by 50^m on the average, and that once in a lunation it does not cross his meridian at all during the mean solar day. The exact difference in the times of transit at Greenwich is obtained by subtracting the time of transit on one day from the time of transit on the next. This difference varies between 42^m and 65^m, and it is given with the times of transits.

The time of upper transit at Greenwich on the 3rd April 1937 is, for example, 5^h27^m. On the 4th April it is 6^h16^m. The difference is 49^m.

Figure 119 shows the Moon and the Mean Sun on the Greenwich meridian at *A*. *B* is a place 75°W. of *A*. When the rotation of the Earth has brought *B* to *A* so that the Mean Sun is on the meridian of *B*, the Moon will have moved along its orbit to *C*.

While the whole 360° of longitude are passing under the Mean

Sun, the Moon reaches D, and AD is a measure of the tabulated daily difference. AC will therefore be to AD in the ratio of the longitude of B to 360°. Thus :

$$\frac{AC}{AD}=\frac{75°}{360°}$$

i.e.

$$AC=\frac{75}{360}\times(\text{the daily difference})$$

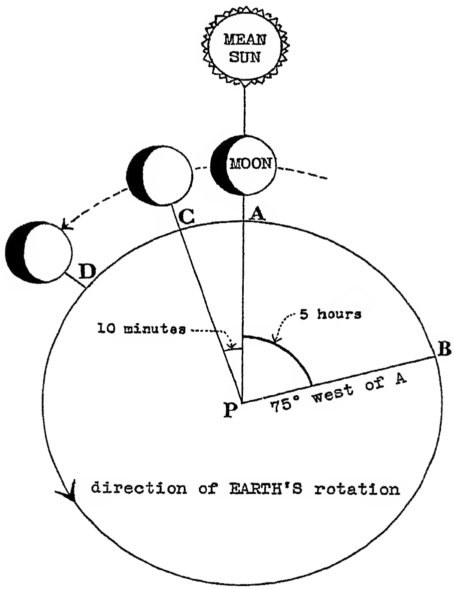


FIGURE 119.

If the daily difference is the 49^m already referred to, the Moon will cross the meridian of 75°W. five hours after it crossed the Greenwich meridian plus an amount :

$$\frac{75}{360}\times 49^m$$
$$=10^m$$

The L.M.T. of the transit at 75°W. will therefore occur 10^m after the L.M.T. of the transit at Greenwich.

If B were 75°E. of A, the 10^m would have to be subtracted because meridian passage would occur at B before it occurred at A, and the daily difference would have to be taken for the day in question and the previous day.

The reason for this can be seen by considering the following

table, which shows the local mean times of transits against the principal meridians.

Date and Daily Difference	*Meridian*	*L.M.T. of Transit* h	m
2nd April	Greenwich	**04**	**37**
	90°W.	04	$49\frac{1}{2}$
(difference 50^m)	180°E. (or W.)	05	02
	90°E.	05	$14\frac{1}{2}$
3rd April	Greenwich	**05**	**27**
	90°W.	05	$39\frac{1}{4}$
(difference 49^m)	180°W. (or E.)	05	$51\frac{1}{2}$
	90°E.	06	$03\frac{3}{4}$
4th April	Greenwich	**06**	**16**

When transits are found with reference to the transit across the Greenwich meridian on the 3rd April, the daily difference involved is that between the 2nd and 3rd for east longitude, and the 3rd or 4th for west longitude. Hence the rules :

(1) At a place in *west* longitude, meridian passage occurs *after* the transit at Greenwich, and a proportion of the difference between the times of the phenomenon on the day in question and the next following is *added* to the L.M.T. at Greenwich to give the L.M.T. on the observer's meridian.

(2) At a place in *east* longitude, meridian passage occurs *before* the transit at Greenwich, and a proportion of the difference between the times of the phenomenon on the day in question and the preceding day is *subtracted* from the L.M.T. at Greenwich to give the L.M.T. on the observer's meridian.

(3) The proportion to be added or subtracted is always :

$$\frac{\text{longitude in degrees}}{360^\circ}\times(\text{daily difference})$$

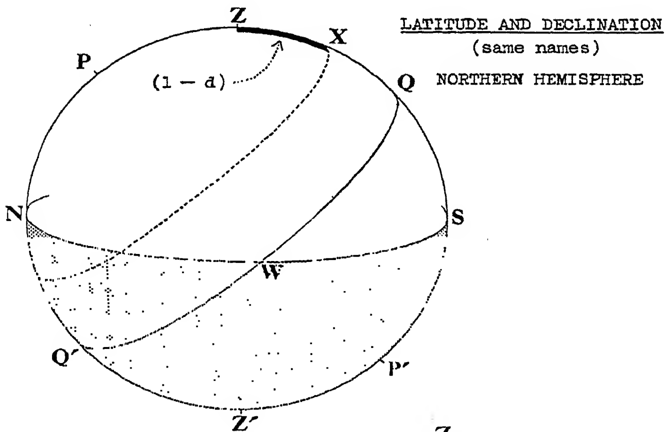
Zone Time of Moon's Transit. The zone time of the Moon's transit across any meridian can be found by converting the L.M.T. of the phenomenon to G.M.T. according to the ordinary rule : longitude west, Greenwich time best ; longitude east, Greenwich time least. Thus :

(1) *What is the zone time of the Moon's meridian passage on the 3rd April* 1937 *in* 64°45′*W.?*

	h m	
L.M.T. of transit at Greenwich	5 27	3rd April
Proportion of Difference ($64\frac{3}{4}\times 49\div 360$)	+9	
L.M.T. of transit at 64°45′W.	5 36	3rd April
Longitude W.	4 19	
G.M.T. of transit at 64°45′W.	9 55	3rd April
Zone (+4)	−4	
∴ Z.T.	0555(+4)	3rd April

(2) *What is the zone time of the Moon's meridian passage on the 3rd April 1937 in 128°12′E.?*

	h m	
L.M.T. of transit at Greenwich	5 27	3rd April
Proportion (128×50÷360)	−18	
L.M.T. of transit at 128°12′E.	5 09	3rd April
Longitude E.	8 33	
G.M.T. of transit at 128°12′E.	20 36	2nd April
Zone (−9)	+9	
∴ Z.T.	0536(−9)	3rd April



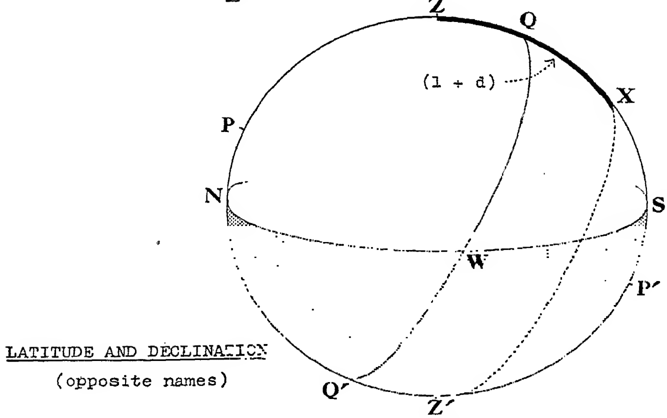


FIGURE 120a. FIGURE 120b.

Altitude at Meridian Passage. Since the altitude at meridian passage is equal to 90° minus the zenith distance at meridian passage, the problem is to find this zenith distance.

In figure 120a, where *the latitude is greater than the declination and of the same name, PQ* and *ZS* are each equal to 90°. Hence :

$$=\text{latitude}-\text{declination}$$

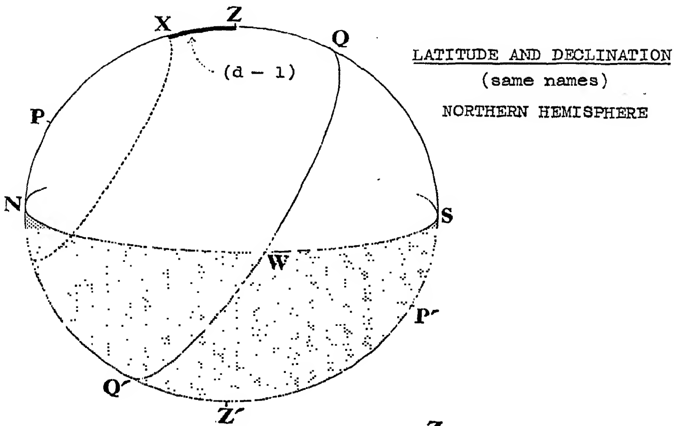


FIGURE 121a.

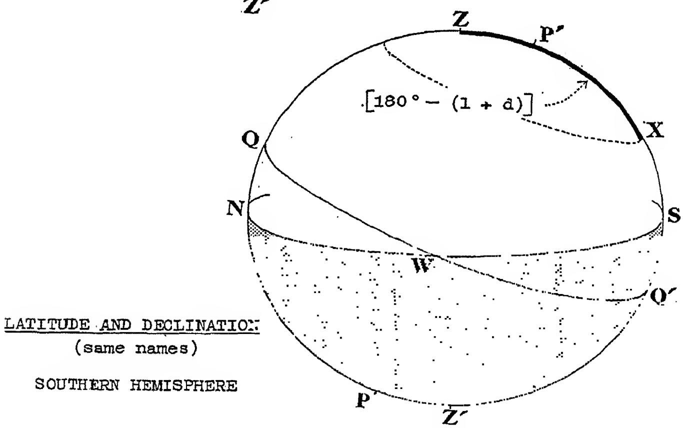


FIGURE 121b.

In figure 120b, where *the latitude and declination have opposite names :*

$$ZX=QZ+QX$$
$$=\text{latitude}+\text{declination}$$

In figure 121a, where *the declination is greater than the latitude and of the same name,* the zenith distance is clearly given by :

$$ZX = \text{declination} - \text{latitude}.$$

That is, in combination :

$$\text{meridian zenith distance} = \text{latitude} \sim \text{declination}.$$

When the transit takes place below the pole (figure 121b) :

$$ZX = 180° - (\text{latitude} + \text{declination})$$

The altitude at upper transit is therefore :

$$90° - (l \sim d)$$

—and at lower transit :

$$(l + d) - 90°$$

Position Line from a Meridian Altitude. The actual position line is found from a meridian altitude exactly as it is found from any other altitude. The advantage of the meridian altitude is that it avoids the labour of calculation. The navigator chooses a position from which to work his sight, and the distance of this chosen position from the geographical position of a heavenly body bearing due north or south is clearly $(l \sim d)$, or, if it is below the pole, $[180° - (l + d)]$. For example :

At Z.T. 1227(+1) *on the 3rd April* 1937, *in D.R.* 51°15′*N.*, 20°48′*W.*, *the sextant meridian altitude of the Sun's lower limb was* 43°56′·8. *The index error was* —0′·5 *and the height of eye* 48 *feet. What was the latitude ?*

Z.T.	1227 3rd April
Zone	+1
G.D.	1327 3rd April

This Greenwich date, which is approximately G.M.T., shows that the Sun's declination is 5°16′·9N. Then :

Sext. Alt.	43°56′·8	Lat. N.	51°15′·0
I.E.	—0′·5	Dec. N.	5°16′·9
Obs. Alt.	43°56′·3	C.Z.D.	45°58′·1
Corr^n.	+8′·3	T.Z.D.	45°55′·4
True Alt.	44°04′·6	Intercept	2′·7 towards
T.Z.D.	45°55′·4		

The Sun's true bearing is 180° since the Sun crosses the meridian to the south of the observer. The latitude is therefore 51°12′·3N., and the position line would appear on the plotting chart as shown in figure 122.

It is, however, not always necessary to find an intercept. If the position line is to be plotted on a Mercator chart, it can be drawn along the parallel without reference to the D.R. position, because the latitude can be found at once by drawing a figure on the plane of the horizon and combining the declination with the true zenith distance.

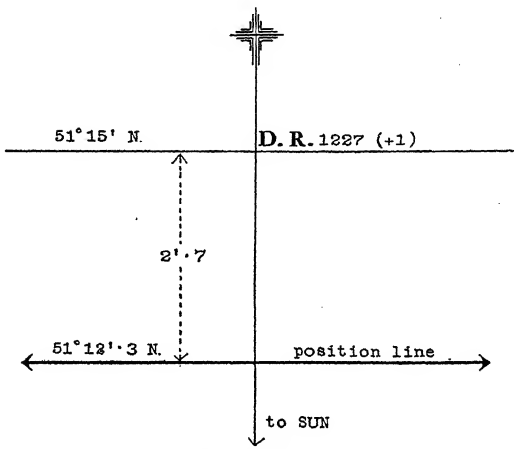


FIGURE 122.

Figure 123 shows the relative positions of the principal points in the last example.

The observer's latitude is given by :

$$QZ = QX + XZ$$
$$= \text{declination} + \text{T.Z.D.}$$

Hence, to find the latitude :

T.Z.D.	45°55′·4
Dec. N.	5°16′·9
Lat. N.	51°12′·3

The figure ensures that the declination is not applied the wrong way.

Meridian Passage during a Given Period. Planets and stars that cross the meridian during a given period—evening twilight, say—can be most conveniently found by means of the star globe described on page 214. But if one is not available, they can be found by considering their right ascensions. For example :

What planets and stars above third magnitude cross the meridian of an observer in 60°*N.*, 45°*W.*, *on the* 3*rd April* 1937, *above and*

below the pole, between 17^h *and* 18^h *L.M.T., and what are their altitudes?*

	h m		h m
L.M.T.	17 00	3rd April	18 00
Long. W.	3 00		3 00
G.M.T.	20 00	3rd April	21 00

For these Greenwich times, take out the values of R to the nearest minute and find the right ascension of the meridian.

	h m	h m
L.M.T.	17 00	18 00
R	12 47	12 47
R.A.M.	5 47	6 47

The right ascension of the observer's meridian during the period under consideration therefore changes from 5^h47^m to 6^h47^m, and any heavenly body that has a right ascension lying between these limits must cross his meridian.

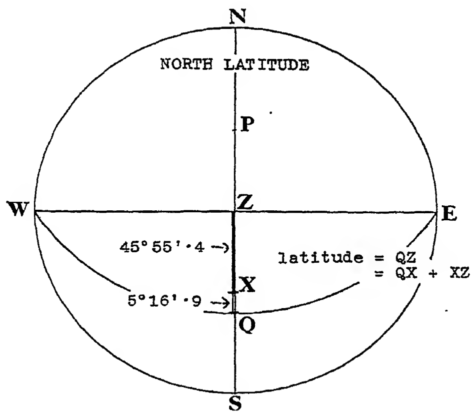


FIGURE 123.

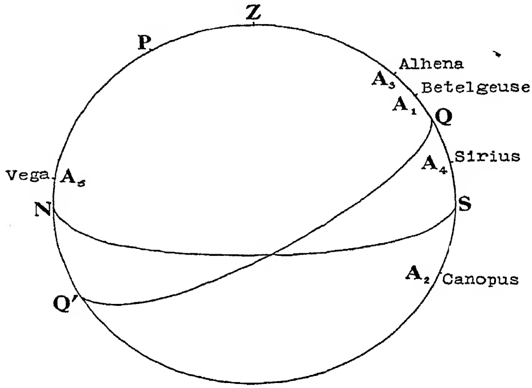
The list of stars for the month of April shows that :

			h m s
Betelgeuse	mag : ·05–1·1	R.A.	5 51 47
Canopus	mag : —0·9	R.A.	6 22 33
Alhena	mag : 1·9	R.A.	6 34 06
Sirius	mag : —1·6	R.A.	6 42 24

—all cross the meridian above the pole. The planet tables show that no planet crosses.

NOTE. The magnitude of a star is the measure of its relative brightness. The term is explained fully in Chapter XXI, which deals with the identification of planets and stars.

To find what planets and stars cross below the pole, 12 hours must be added to the limiting right ascensions. These become 17^h47^m and 18^h47^m, and the tables show that only Vega (mag : 0·1 ; R.A. $18^h34^m50^s$) crosses below the pole.



For convenience in drawing, the stars in this figure are shown as if their transits occurred simultaneously. Actually these transits are spread over an hour.

FIGURE 124.

The declinations of these five stars are :

Betelgeuse	7°23′·7N.
Canopus	52°40′·0S.
Alhena	16°27′·2N.
Sirius	16°38′·0S.
Vega	38°43′·2N.

The meridian altitudes, from figure 124, are :

Betelgeuse : $SA_1=SQ+QA_1$
$=30°+7°23'·7$
$=37°23'·7$

Canopus : Clearly a distance SA_2 below the horizon, where
$SA_2=QA_2-QS$
$=52°40'-30°$
$=22°40'$

Alhena : $SA_3=SQ+QA_3$
$=3$
$=46°27'·2$

Sirius $SA_4=SQ-QA_4$
$=30°-16°38'$
$=13°22'$

Vega $NA_5=Q'A_5-Q'N$
$=38°43'·2-30$
$=8°43'·2$

All the stars, except Canopus, are therefore visible at meridian passage.

CHAPTER XVIII

EX-MERIDIAN TABLES

For hour angles less than $2^h 10^m$, and for certain limits of latitude and declination, the spherical triangle can be solved by means of special tables known as ex-meridian tables. The principle involved is known as a ' reduction to the meridian ', and the limits referred to are the limits of the tables themselves.

From these tables is found a small quantity that, when added to $(l \sim d)$ or $[180° - (l + d)]$, gives the calculated zenith distance and

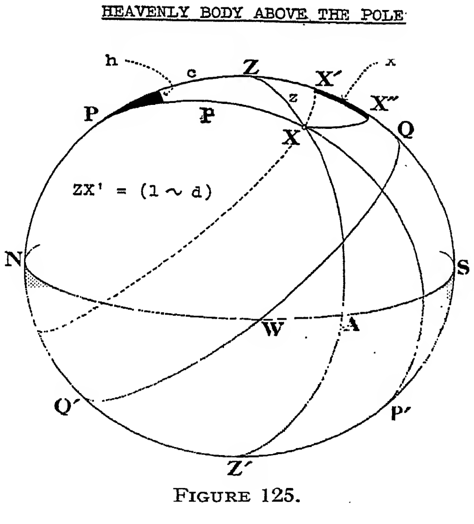


FIGURE 125.

therefore an intercept. The tabıes thus do no more than provide another method of finding the C.Z.D., and it is not necessary to use them merely because the hour angle is small.

Construction of the Ex-Meridian Tables. Figure 125 illustrates the principle upon which the tables are built.

AX is the altitude of the heavenly body when the hour angle is h. SX' is its altitude at meridian passage. XX' is parallel to the celestial equator. XX'' is parallel to the horizon.

Therefore SX'' is equal to AX, and $X'X''$ is the small amount, x, that must be added to the altitude of the heavenly body when the hour angle is h in order to give the altitude at meridian passage.

When the heavenly body is on the meridian, the calculated zenith distance (ZX') is equal to ($l \sim d$). When the heavenly body is near the meridian, the calculated zenith distance is $[(l \sim d)+x]$ because ZX (which is equal to ZX'') is greater than ZX' by the small amount x.

In the construction of the tables, all quantities are referred to the spherical triangle PZX. Thus:

$$\begin{aligned} l &= 90° - c \\ d &= 90° - p \\ z &= (l \sim d) + x \\ &= (90° - c) \sim (90° - p) + x \\ &= (p \sim c) + x \end{aligned}$$

The fundamental formula therefore becomes:

$$\cos [(p \sim c) + x] = \cos p \cos c + \sin p \sin c \cos h$$

In this equation x is the only unknown; c is assumed; p is taken (as d) from the *Nautical Almanac*, and h is calculated from a deck-watch time. The small quantity x is thus found in terms of the latitude, declination and the hour angle, and these are the arguments used in the tables.

Table I. This has latitude and declination for arguments, and is built in three parts: one for latitudes and declinations that have the same names, one for those having opposite names, and one for observations made when the heavenly body is near the meridian below the pole.

Table II. This has the hour angle for argument, and is the ordinary logarithmic haversine table given to three places of decimals instead of the usual five.

Table III. This gives an approximate value of the small quantity x. The body of the table is entered and against the number that is the sum of the quantities taken out of Tables I and II, the approximation is read in minutes of arc at the side and in decimals of minutes at the top.

Table IV. This gives a correction which must be subtracted from the approximation found in Table III. The arguments are this first approximation and the altitude.

All these tables are included in *Inman's Tables*, and their full explanation is given in Volume III.

The following examples illustrate the methods of using the tables for observations near upper and lower meridian passages.

Ex-Meridian Observation Above the Pole. *At Z.T.* 1150(+1) *on the 3rd April* 1937, *in D.R.* 41°36′*N.*, 21°18′*W.*, *the deck watch showed* $0^h50^m18^s$ *when the sextant altitude of the Sun's lower limb was* 52°34′·4. *The index error was* +1′·3 ; *the height of eye* 40 *feet* ; *and the deck watch was* 12^s *slow on G.M.T.*

Z.T.	1150 3rd April	
Zone	+1	
G.D.	1250 3rd April	

Declination	5°15′·5N.
	+0′·8
	5°16′·3

	h m s	
D.W.T.	0 50 18	
Error slow	12	
G.M.T.	12 50 30	3rd April
Long. W.	1 25 12	
L.M.T.	11 25 18	
E	11 56 36	
H.A.T.S.	23 21 54	

E
h m s
11 56 35·2
+0·6
11 56 35·8

From the tables :

Table I (for lat. 41°36′N., dec. 5°16′·3N.)		·100
Table II (for H.A.T.S. $23^h21^m54^s$)		7·838
	(sum)	7·938
Table III (for 7·938)		59′·6
Table IV (for 59′·6 and altitude 52°34′·4)		—·7
	∴ x=	58′·9

Lat. N.	41°36′·0	Sext. Alt.	52°34′·4
Dec. N.	5°16′·3	I.E.	+1′·3
$l \sim d$	36°19′·7	Obs. Alt.	52°35′·7
$\sim$	+58′·9	Corrn.	+9′·2
C.Z.D.	37°18′·6	True Alt.	52°44′·9
T.Z.D.	37°15′·1	T.Z.D.	37°15′·1
Intercept	3′·5 towards		

From the tables the true bearing is 164°.
The position line is therefore as shown in figure 126.

Ex-Meridian Observation Below the Pole. *At Z.T.* 0410(+2) *on the 3rd April* 1937, *in D.R.* 60°28′*N.*, 36°12′*W.*, *the deck watch showed* $6^h07^m43^s$ *when the sextant altitude of Capella was* 16°56′·2. *The index error was* —1·4 ; *the height of eye* 30 *feet ; and the deck watch was* 5^s *fast on G.M.T.*

Z.T.	0410 3rd April		Declination
Zone	+2		45°56′·3N.
G.D.	0610 3rd April		R
	h m s		h m s
D.W.T.	6 07 43		12 44 36·5
Error fast	05		+1·3
G.M.T.	6 07 38	3rd April	12 44 37·8
Long. W.	2 24 48		
L.M.T.	3 42 50		
R	12 44 38		
R.A.M.	16 27 28		
R.A.	5 12 03		
H.A.	11 15 25		

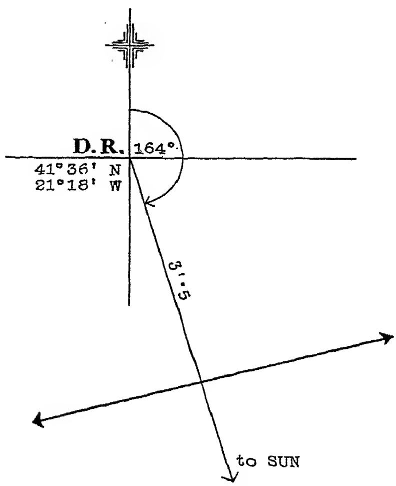


FIGURE 126.

The rules for using the tables when the heavenly body is below the pole are :

(1) Work with the angle at *P* measured from the observer's lower meridian—in this example ($12^h—11^h15^m25^s$) or $0^h44^m35^s$.

(2) Change the sign of the second correction and subtract the total correction from $[180°-(l+d)]$.

Figure 127 explains these rules. Applied to this example they give :

Table I (for lat. 60°28′N., dec. 45°56′·3N.)		9·553
Table II (for H.A. $0^h 44^m 35^s$)		7·975
	(sum)	7·528
Table III (for 7·528)		23′·2
Table IV (for 23′·2 and altitude 16°56′·2)		0′·0
	∴ $x=$	23′·2

Lat. N.	60°28′·0	Sext. Alt.	16°56′·2
Dec. N.	45°56′·3	I.E.	−1′·4
$180°-(l+d)$	73°35′·7	Obs. Alt.	16°54′·8
x	23′·2	Corr^n.	−8′·6
C.Z.D.	73°12′·5	True Alt.	16°46′·2
T.Z.D.	73°13′·8	T.Z.D.	73°13′·8
Intercept	1′·3 away		

HEAVENLY BODY BELOW THE POLE

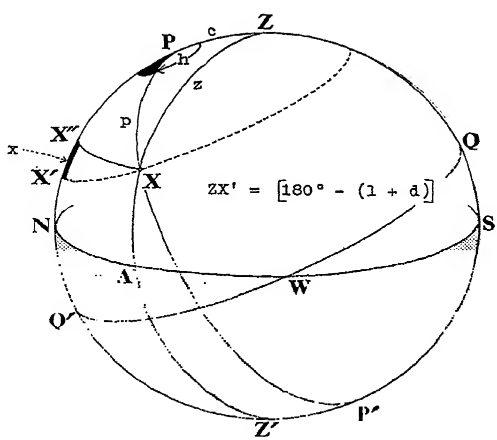


FIGURE 127.

By calculation the azimuth of Capella is N.8°W. The true bearing is therefore 352°.

Figure 128 shows the position line.

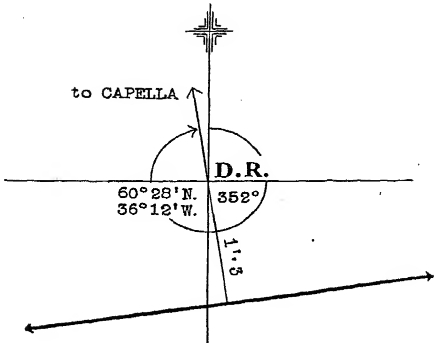


FIGURE 128.

The above examples should be followed carefully with the actual tables, because the real drawback of the method, which lies in the awkward interpolation necessary with Table I, will then be apparent.

CHAPTER XIX

THE POLE STAR

Polaris or the Pole Star is the name given to the second magnitude star which lies close to the north celestial pole in the celestial sphere. If its position coincided with the north celestial pole, the problem of finding the latitude from an observation of it would be easy because, as shown on page 73, the altitude of the pole is equal to the latitude of the observer. But the declination of the Pole Star, instead of being 90°N. as it would have to be for coincidence with the north celestial pole, is approximately 89°N. The polar distance is thus approximately 1°, and in the course of a day the Pole Star describes a small circle about the pole with that angular radius. The altitude of the Pole Star is thus not quite equal to the latitude of the observer.

Figure 129 shows this small circle of angular radius p.

Latitude by the Pole Star. When the Pole Star is at some position X, its altitude is AX and the observer's latitude PN is given by :

$$PN = AX \pm PY$$

—where the position of Y is such that ZY is equal to ZX. For the situation depicted in figure 129, the minus sign must be taken.

The problem is therefore to find PY for any hour angle, because the length of PY clearly depends on the position of X, and the position of X depends on the hour angle.

The Pole Star Tables. Since the angular radius of the circle described about P is small, the arc XY approximates closely to the perpendicular XY', and the right-angled triangle PXY' is sufficiently small to be considered plane. Hence it follows that, as a first approximation :

$$PY = p \cos h$$

—where h is the hour angle of the Pole Star.

Table I gives this first approximation.

Since h is equal to (R.A.M. − R.A.✱), that is to (R.A.M. − a constant), a step is saved by entering the table with the R.A.M. or local sidereal time as the argument.

Table II gives a correction to this first approximation, based on the formula to be found in the explanation of the tables in the *Nautical Almanac*. This formula is also explained fully in Volume III. The correction itself is, in brief, deduced from a consideration of the spherical as opposed to the plane triangle.

Table III corrects Table I for the difference between the true right ascension and declination of the Pole Star at any date, and the quantities assumed for building Table I. The arguments are local sidereal time and date.

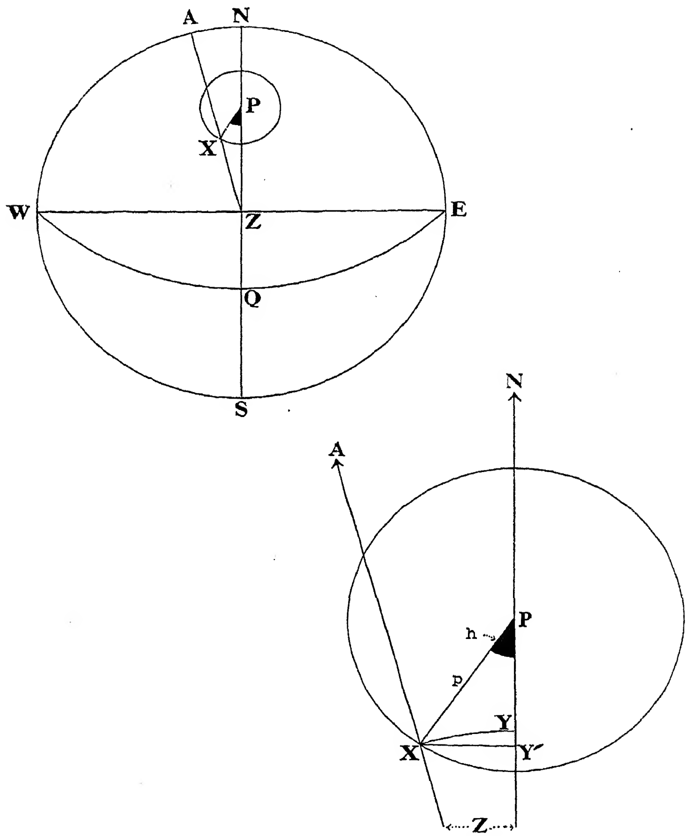


FIGURE 129.

Procedure for Using the Pole Star Tables. This procedure may be set out in the following steps :

(1) Correct the sextant altitude in the same way that the altitude of any other star is corrected.

(2) From the deck watch time work out the R.A.M. Great accuracy is not required since the maximum difference over a two-minute interval in Table I is only 0′·6. It is therefore sufficient to work to the nearest minute.

(3) With this R.A.M., enter Table I, and apply the correction obtained to the true altitude according to the sign given in the table.

(4) With the altitude and the R.A.M. enter Table II and *add* this correction to the quantity arrived at by the previous step.

(5) With the date and the R.A.M. enter Table III and apply the correction according to its sign to the quantity arrived at by step (4). The result is the observer's latitude.

Because the azimuth of the Pole Star in latitudes suitable for observing the star itself does not exceed $2\frac{1}{2}°$, the position line may be taken as lying along the parallel of latitude.

At Z.T. 1905(+3) *on the 3rd April* 1937, *in D.R.* 46°15′*N.*, 37°32′*W.*, *the deck watch showed* $10^h04^m39^s$ *when the sextant altitude of the Pole Star was* 46°18′·2. *The index error was* —1′·8 ; *the height of eye* 32 *feet, and the deck watch was* 4^s *slow on G.M.T.*

Z.T	1905 3rd April	
Zone	+3	
G.D.	2205 3rd April	

	h m s	
D.W.T.	10 04 39	
Error slow	4	
G.M.T.	22 05	— 3rd April (to nearest minute)
Long. W.	2 30	
L.M.T.	19 35	
R	12 47	
R.A.M.	8 22	

Sext. Alt.	46°18′·2	
I.E.	— 1′·8	
Obs. Alt.	46°16′·4	
Corr^n.	— 6′·6	
True Alt.	46°09′·8	
Table I	+11′·3	
Table II	+ 0′·5	
Table III	0′·0	
∴ Latitude	46°21′·6	

The position line is therefore as shown in figure 130.

Observation of the Pole Star at Twilight. Since the Pole Star is not a particularly bright star, it does not appear to the naked eye until the horizon has become indistinct in the gathering dusk. An effective way of overcoming this difficulty is therefore to apply to the D.R. latitude the Table I correction with the sign changed, and put this approximate altitude on the sextant. This will enable the star to be found in the telescope, and an observation to be taken with a good horizon, long before the naked eye can detect the star.

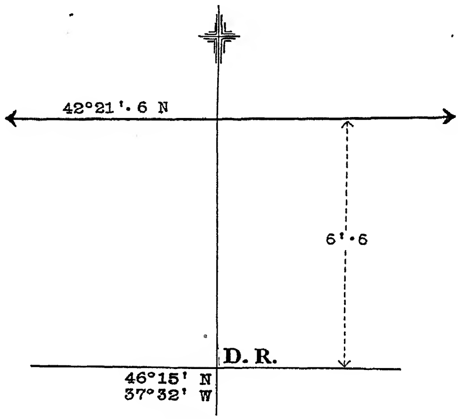


FIGURE 130.

Azimuth of the Pole Star. The *Nautical Almanac* gives a table of azimuths for latitudes from 10° to 60°, because these azimuths offer a ready means of checking the error of the compass.

The azimuth of the Pole Star in the last example is N.1°·4W. That is, its true bearing is, for all practical purposes, 358½°. If, at the moment the sight was taken, the Pole Star bore 359° by compass, the compass would be reading ½° high.

CHAPTER XX

THE OBSERVED POSITION

In the explanation of the terrestrial position line given in Chapter VI, it was stated that the use to which a position line can be put is independent of the source from which the line itself is obtained because a position line does no more than tell the navigator that his position lies somewhere on it. A navigational fix requires at least two position lines.

The examples worked in this chapter are designed to show how the position of a moving ship can be found by observations of those heavenly bodies most familiar to navigators.

The reliance that can be placed upon the position obtained by a particular combination depends on a number of factors: the method by which the sights are worked, the accuracy of the almanac from which the astronomical quantities are taken, the precision with which the altitudes are measured, and the error involved in the estimation of any run there may have been between sights. Some of these factors have been referred to already. All are discussed in Volume III. By taking 'simultaneous' sights during twilight the navigator clearly avoids the error that a run between sights may introduce, and, in addition, he has the satisfaction of obtaining his position without delay. It should be remembered, however, that in practice sights are, at times, taken when the opportunities occur.

MOON—RUN—SUN

3rd April 1937. *Course* 070°. *Speed* 12 *knots. At about* 0500(+1) *in estimated position* 50°10′*N.*, 14°50′*W.*, *the sextant altitude of the Moon's lower limb was* 16°25′·0 *when the deck watch showed* $5^h59^m40^s$.

At about 0740(+1), *the sextant altitude of the Sun's lower limb was* 19°48′·0 *when the deck watch showed* $8^h40^m05^s$.

The index error was +1′·5 ; *the height of eye* 40 *ft.* ; *and the deck watch was* 5^s *slow on G.M.T. The Moon's horizontal parallax was* 54′·8.

Required the ship's observed position at 0740(+1).

This is a straightforward and common type of double sight in which the first position line is transferred for the run between sights exactly as the terrestrial position line was transferred in the example on page 61.

To Find the First Position Line

Z.T.	0500 3rd April		
Zone	+1		
G.D.	0600 3rd April		
	h m s		
D.W.T.	5 59 40		
Error slow	5		
G.M.T.	5 59 45 3rd April		
Long. W.	59 20		
L.M.T.	5 00 25		
R	12 44 36·5		
R.A.M.	17 45 01·5		
R.A.	18 12 22·5		
H.A.	23 32 39	7·550	97
Lat. N.	50°10′·0	9·806	56
Dec. S.	22°02′·4	9·967	04
	72°12′·4	7·324	57
		·002	11
		·347	21
	C.Z.D. = 72°27′·7	·349	32

From the *Nautical Almanac*

☾'s H.P.	—	54′·8
☾'s Dec.	—	22°02′·4S.
☾'s R.A.	—	$18^h 08^m 04^s·0$
		4 18 ·5
		18 12 22 ·5
R	—	$12^h 44^m 36^s·5$

Sext. Alt.	16°25′·0
I.E.	+1′·5
Obs. Alt.	16°26′·5
Corr^n.	+58′·2
True Alt.	17°24′·7
T.Z.D.	72°35′·3
C.Z.D.	72°27′·7
Intercept	·6 away
True Bearing	173°

The observed position of the ship at 0740 can be obtained by three methods, and, if necessary, by combinations of them. These methods are :

(1) calculation with the traverse table.
(2) drawing on a plotting chart.
(3) direct plotting on a Mercator chart.

The Observed Position by Traverse Table. In order that the first position line may be correctly transferred, the estimated position of the

ship at 0740 must be found as accurately as possible. In this example, the only information on which to work is that relating to the ship's course and speed. The subsequent plotting, however, is simplified without any loss of accuracy if the ship's run is continued, not from the first D.R. or E.P., but from the point J on the first intercept

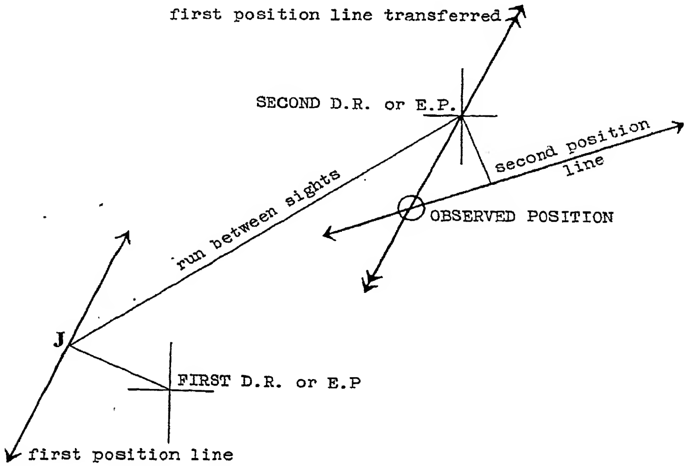


FIGURE 131.

through which the position line is drawn. Figure 131 illustrates the principle.

The result is that the first intercept can be incorporated in the second E.P., and the first position line can be transferred so as to pass through the second E.P.

To Find the Estimated Position at 0740(+1)

The run from 0500(+1) to 0740(+1) is 32′—2^h40^m at 12 knots—at 070°.

By traverse table:

		d'lat N.	d'lat S.	dep. E.	dep. W.
Intercept	7′·6 at N.7°W.	7′·5			0′·9
Run	32′ at N.70°E.	10′·9		30′·1	
		18′·4N.		29′·2E.	

Hence:

mean lat. = 50°19′·2N.
d'long = 45′·7E.

The estimated position at 0740(+1) is therefore $\begin{cases} 50°28'·4\text{N.} \\ 14°04'·3\text{W.} \end{cases}$

To Find the Second Position Line

Z.T.	0740 3rd April	
Zone	+1	
G.D.	0840 3rd April	

	h m s		
D.W.T.	8 40 05		
Error slow	5		
G.M.T.	8 40 10	3rd April	
Long. W.	56 17·2		
L.M.T.	7 43 52·8		
E	11 56 32·7		
H.A.T.S.	19 40 25·5		9·459 17
Lat. N.	50°28′·4		9·803 76
Dec. N.	5°12′·3		9·998 21
	45°16′·1		9·261 14
			·182 45
			·148 11
∴ C.Z.D.=70°11′·5			·330 56

From the *Nautical Almanac*
E
h m s
11 56 32·2
0·5
11 56 32·7
Sun's Dec.
5°11′·7N.
0′·6
5°12′·3N.

Sext. Alt.	19°48′·0
Index Error	+1′·5
Obs. Alt.	19°49′·5
Corr^{n}.	+7′·2
True Alt.	19°56′·7
T.Z.D.	70°03′·3
C.Z.D.	70°11′·5
Intercept	8′·2 towards
True Bearing	106½°

To Combine the Two Position Lines

Figure 132 shows the transferred position *OB* and the second position line *JB*, cutting at *B*, the observed position. The distances required are *AB*, the d'lat, and *OA*, the departure, and clearly

it is easier to measure these than to calculate them from the right-angled triangle OJB by means of the traverse table. This method, if followed, would give, since OJ is $8'{\cdot}2$ and the angle BOJ is $23\frac{1}{2}°$:

$$\begin{aligned} OB &= 9'{\cdot}0 \\ \therefore \quad \text{d'lat} &= 1'{\cdot}1\text{N.} \\ \text{departure} &= 8'{\cdot}9\text{E.} \end{aligned}\Bigg\}\text{from triangle } AOB$$

$$\text{d'long} = 14'{\cdot}0\text{E.}$$

The ship's observed position at 0740(+1) is therefore $\begin{cases} 50°29'{\cdot}5\text{N.} \\ 13°50'{\cdot}3\text{W.} \end{cases}$

The Observed Position by Plotting Chart. A drawing on a plotting chart is simply a graphical representation of traverse-table work.

Figure 133 shows the complete plot, and the d'lat ($AB=19'{\cdot}5$N.) and the departure ($OA=38'$E.) are found at once. The d'long is therefore $59'{\cdot}7$E. and, as before, the observed position at 0740(+1) is :

$$\begin{aligned} &50°29'{\cdot}5\text{N.} \\ &13°50'{\cdot}3\text{W.} \end{aligned}$$

A probable combination of this method and the previous would be :

(1) calculation of the 0740 E.P.

(2) plotting of the observed position from this.

The Observed Position by Mercator Chart. In practice a Mercator chart is usually available, and it affords by far the quickest method of obtaining the observed position because longitude can be read directly without reference to departure.

Distances are measured on the latitude scale level with that part of the chart covered by the run between sights, and the actual plotting is in no way different from that which is done on a plotting chart.

Procedure when Sights are Worked from Special Positions. When a tabular method is used, the intercept is laid off from a special position which may be as much as forty miles from the D.R., and therefore a like distance from the ship's actual position, which should be close to the D.R. The point on this intercept through which the position line is drawn is thus, in all probability, some distance from her actual position, and if her track is shown on the chart as passing through this point, it will convey an erroneous idea of position. For this reason *a ship's track should always be run from the point on the position line nearest the D.R.*, and that is the point through which the position line is drawn when the sight is worked from the D.R. or E.P. Tabular methods are thus essentially practical methods for use with the Mercator chart.

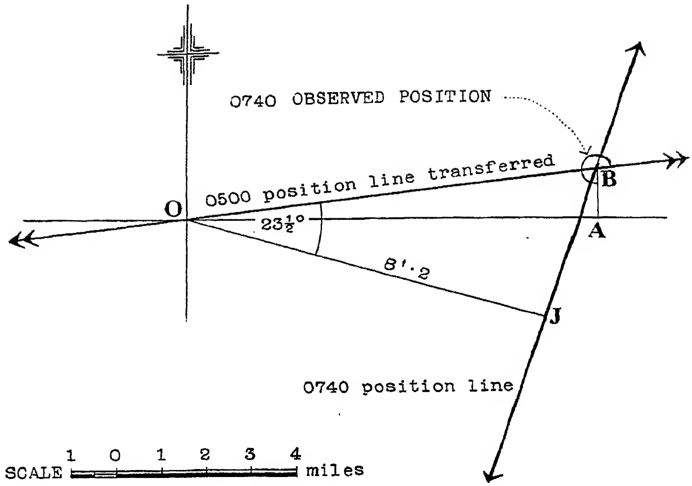


FIGURE 132.

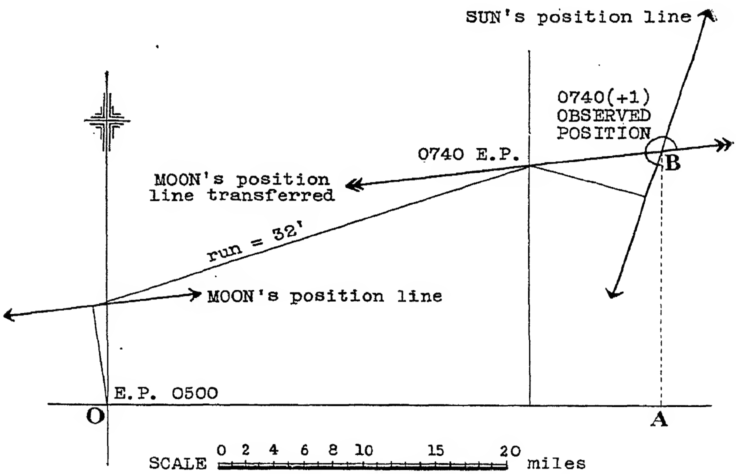


FIGURE 133.

The procedure by which a navigator using the S.A.N. Tables, for example, would deal with the Moon and Sun sights which have been worked by the cosine-haversine method, would be :

Moon Sight

Z.T.	0500 3rd April
Zone	+1
G.D.	0600 3rd April

	h m s
D.W.T.	5 59 40
Error slow	5
G.M.T.	5 59 45
R	12 44 36·5
R.A.M.(G.)	18 44 21·5
R.A.	18 12 22·5
G.H.A.	0 31 59
=	7°59′·7
Long. W.	14°59′·7
H.A. ☾	’W. or 7°E.

☾’s H.P. —	54′·8
☾’s Dec. —	22°02′·4S.
☾’s R.A. —	$18^h08^m04^s·0$
	4 18 ·5
	18 12 22 ·5
R —	$12^h44^m36^s·5$

Sext. Alt.	16°25′·0
I.E.	+ 1′·5
Obs. Alt.	16°26′·5
Corr^n.	+58′·2
True Alt.	17°24′·7

Assumed latitude 50°N.

Dec.	22°02′·4S.						
K	50°12′·7N.	A	134	D	1106	Z_1	+84°·6
$K \sim d$	72°15′·1	B	51593	E	495		
		A+B	51727	D+E	1601	Z_2	+88°·6
							Az. N.173°·2E.

Calc. Alt.	17°41′·5
True Alt.	17°24′·7
Intercept	16′·8 away

The navigator lays off this intercept from a position 50°N., 14°59′·7W. on the Mercator chart, draws the position line—it is identical with that obtained by the cosine-haversine method—and continues the ship’s track from the point on it nearest the 0500 E.P.

By marking off a run of 32′ along this track he obtains his 0740 E.P. and notes that it is 50°28′·4N., 14°04′·3W.

Sun Sight

		E
		h m s
Z.T.	0740 3rd April	11 56 32·2
Zone	+1	0·5
G.D.	0840 3rd April	11 56 32·7

	h m s		
D.W.T.	8 40 05	Sun's Dec.	
Error slow	5	5°11′·7N.	
		0′·6	
G.M.T.	8 40 10 3rd April		
E	11 56 32·7	5°12′·3N.	
G.H.A.	20 36 42·7		
=	309°10′·7	Sext. Alt.	19°48′·0
Long. W.	14°10′·7	I.E.	+1′·5
H.A.T.S.	295°W. or 65°E.	Obs. Alt.	19°49′·5
		Corrⁿ.	+7′·2
		True Alt.	19°56′·7

Assumed latitude	50°N.						
Dec.	5°12′·3N.						
K	70°28′·5N.	A	9003	D	235	Z_1	+31°·3
	65°16′·2	B	37847	E	337		
		A+B	46850	D+E	572	Z_2	+75°·0
							Az. N.106°·3E.

Calc. Alt.	19°52′·7
True Alt.	19°56′·7
Intercept	4′·0 towards

The navigator then lays off this intercept from a position 50°N., 14°10′·7W., draws the position line, which is again identical with that obtained by the cosine-haversine method, transfers the first position line to pass through the 1740 E.P. (*not through the position from which he has worked the second sight*) and notes that the point where they cut, his observed position, is 50°29′·5N., 13°50′·3W.

Figure 134 shows the plot as it would appear on the chart. It should be compared with figure 133.

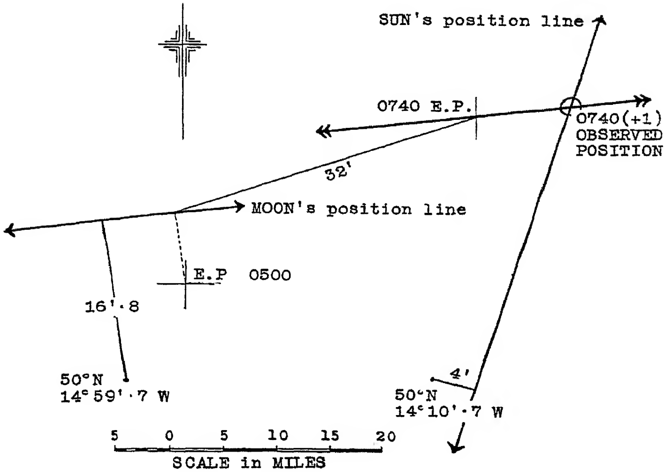


FIGURE 134.

SIMULTANEOUS STAR AND PLANET SIGHTS

The word ' simultaneous ' is commonly applied to sights which are taken at approximately the same time and, for that reason, can be worked from the same D.R. This does not mean, however, that the interval which elapses between the observations of two separate stars can be ignored. If the ship is stopped or moving slowly so that her run between sights is negligible, then clearly no adjustment is necessary. But a fast-moving ship may cover an appreciable distance between sights, and the first position line must be ' run on ' accordingly, or both position lines adjusted to a particular time, in order to obtain an accurate observed position. The next example shows how this adjustment is made.

3rd April 1937. *Course* 030°. *Speed* 25 *knots. At about* 1730(—8) *in estimated position* 20°10'*N.*, 120°40'*E.*, *the following observations were obtained :*

	D.W.T.	*Sext. Alt.*	*Approx. Bearing*	
	h m s			
Polaris	9 20 04	20°37'·3	360°	From
Mirfak	9 24 10	45°04'·6	320°	Star
Venus	9 29 18	30°11'·5	280°	Globe

The index error was +2'·5, *the height of eye* 24 *feet ; and the deck watch was* 12ˢ *slow on G.M.T.*

Required the ship's position at 1730(−8).

		R.A. *Venus*	Dec.
		h m s	
Z.T.	1730 3rd April	2 00 31	20°09′·5N.
Zone	−8	−27	−1′·7
G.D.	0930 3rd April	2 00 04	20°07′·8N.

	Polaris		*Mirfak*	*Venus*
	h m s		h m s	h m s
D.W.T.	9 20 04		9 24 10	9 29 18
Error	+12		+12	+12
G.M.T. (3rd April)	9 20 16		9 24 22	9 29 30
Long. E.	8 02 40		8 02 40	8 02 40
L.M.T.	17 22 56		17 27 02	17 32 10
R	12 45 09		12 45 10	12 45 11
R.A.M.	6 08 05		6 12 12	6 17 21
R.A.			3 19 50	2 00 04
H.A.			2 52 22	4 17 17
Lat.			20°10′·0N.	20°10′·0N.
Dec.			49°38′·4N.	20°07′·8N.
			29°28′·4	0°02′·2
			9·129 91	9·452 30
			9·972 52	9·972 52
			9·811 30	9·972 63
			8·913 73	9·397 45
			·081 99	·249 72
			·064 71	·000 00
			·146 70	·249 72
Sext. Alt.	20°37′·3		45°04′·6	30°11′·5
I.E.	+2′·5		+2′·5	+2′·5
Obs. Alt.	20°39′·8		45°07′·1	30°14′·0
Corr^{n}.	−7′·3		−5′·8	−6′·5
True Alt.	20°32′·5		45°01′·3	30°07′·5
Tables I & II	−24′·0	T.Z.D.	44°58′·7	59°52′·5
	20°08′·5	C.Z.D.	45°02′·5	59°57′·8
Table III	0′·0	Intercept	3′·8 towards	5′·3 towards
Latitude	20°08′·5	Bearing	321°	282°

The above working is set out in parallel columns according to the practice of navigators, and it is seen that, by doing so, space

and labour are saved and the chance of making an arithmetical mistake is lessened.

Figure 135 shows the plotting of the observed position.

When the intercepts are laid off from the same E.P. and the position lines drawn with no adjustment for time, they do not meet in a point but form the ' cocked hat ' shown by the shaded triangle. To obtain the correct position, the position lines must be ' run on ' to 1730(—8).

From the G.M.T.s of the observations it is seen that Polaris was actually observed at 1720¼(—8). Polaris must therefore be run on for 9¾ minutes at 25 knots, or 4′; Mirfak for 5½ minutes, or 2′·3; and Venus for ½ minute, or 0′·2; the course being 030°.

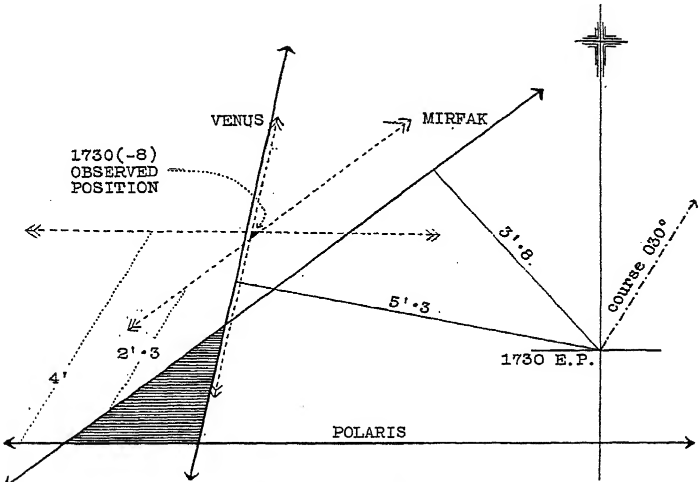


FIGURE 135.

These ' run on ' or transferred position lines are shown by the dotted lines, and it is seen that the cocked hat disappears. The common point of intersection is the observed position.

From the figure :

	d'lat=2′·0N.	dep.=5′·0W.
		d'long=5′·3W.
E.P.	lat. 20°10′N.	long. 120°40′·0E.

The ship's observed position at 1730(—8) is therefore $\left\{\begin{array}{l}20°12′·0N.\\120°34′·7E.\end{array}\right.$

SUN—RUN—MERIDIAN ALTITUDE

3rd April 1937. *Course* 070°. *Speed* 12 *knots. At about* 0830(+2) *in estimated position* 51°15′*N.*, 29°45′*W.*, *the sextant*

altitude of the Sun's lower limb was 26°10′·0 *when the deck watch showed* $10^h29^m50^s$.

At about 1200(+2) *the sextant meridian altitude of the Sun's lower limb was* 43°32′·4.

The index error was +2′·5 ; *the height of eye* 40 *feet ; and the deck watch was* 3^s *slow on G.M.T.*

Required the ship's position at the time of meridian altitude.

Z.T.	0830 3rd April	Sun's Declination
Zone	+2	5°13′·6N.
		0′·5
G.D.	1030 3rd April	
		5°14′·1N

	h m s	
D.W.T.	10 29 50	
Error slow	3	E
		h m s
G.M.T.	10 29 53 3rd April	11 56 33·7
Long. W.	1 59 00	0·4
L.M.T.	8 30 53	11 56 34·1
E	11 56 34	

H.A.T.S.	20 27 27	9·301	14
Lat. N.	51°15′·0	9·796	52
Dec. N.	5°14′·1	9·998	19
$l \sim d$	46°00′·9	9·095	85
		·124	69
		·152	77
∴ C.Z.D. = 63°34′·3		·277	46

Sext. Alt.	26°10′·0
I.E.	+2′·5
Obs. Alt.	26°12′·5
Corr^n.	+8′·0
True Alt.	26°20′·5
T.Z.D.	63°39′·5
C.Z.D.	63°34′·3
Intercept	5′·2 away

True Bearing 117°

The Run between Sights

The run between sights can be found only when the time of meridian passage is known. This time must therefore be calculated by the methods explained in Chapter IV.

For a first approximation the time of meridian passage may be taken as 1200(+2).

Between 0830(+2) and 1200(+2) the ship runs for $3\frac{1}{2}$ hours at 12 knots, a distance of 42′, at 070°.

By traverse table :

	d'lat N.	*d'lat* S.	*dep.* E.	*dep.* W.
Run 42′ at N.70°E.	14′·4		39′·5	
Intercept 5′·2 at N.63°W.	2′·4			4′·6
	16′·8N.		34′·9E.	

mean lat. = 51°23′·4N.
d'long = 56′·1E.

The E.P. at 1200(+2) is therefore $\begin{cases} 51°31'·8\text{N.} \\ 28°49'·1\text{W.} \end{cases}$

The longitude thus obtained enables a more accurate time of meridian passage to be found.

	h m
H.A.T.S.	0 0
E	11 57 (for 14ʰ, G.M.T.)
L.M.T.	12 03
Long. W.	1 55
G.M.T.	13 58 3rd April
Zone (+2)	−2
Z.T.	1158(+2) 3rd April

The E.P. at this moment is found by running back the 1200(+2) position a distance 0′·4, the run in two minutes at 12 knots.

The E.P. at 1158(+2) is therefore $\begin{cases} 51°31'·6\text{N.} \\ 28°49'·7\text{W.} \end{cases}$

To find the Second Position Line

Z.T.	1158 3rd April	Sun's declination	5°17′·4N.
Zone	+2	E.P. latitude	51°31′·6N.
G.D.	1358 3rd April	C.Z.D. = $(l \sim d)$	46°14′·2

Sext. Alt.	43°32′·4
I.E.	+2′·5
Obs. Alt.	43°34′·9
Corrⁿ.	+8′·9
True Alt.	43°43′·8
T.Z.D.	46°16′·2
C.Z.D.	46°14′·2
Intercept	2′·0 away
True Bearing	180°

The latitude is known at once because the second position line coincides with the parallel of latitude. The d'lat is thus 2'N.

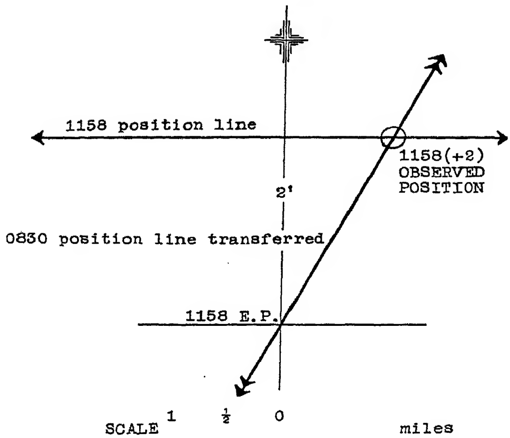


FIGURE 136.

By measurement, the departure is 1'E., and this gives a d'long of 1'·6E.

The observed position at 1158(+2) is therefore $\begin{cases} 51°33'·6N. \\ 28°48'·1W. \end{cases}$

CHAPTER XXI

IDENTIFICATION OF PLANETS AND STARS

To an observer on the Earth, the Sun, the Moon and the planets move against the fixed background of the stars. In addition to the Earth, the Sun is known to have eight large satellites. They are Mercury, Venus, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto, but, as already stated, only four—Venus, Mars, Jupiter and Saturn—are sufficiently bright for navigational purposes. The number of small satellites and asteroids revolving about the Sun is known to be about 1,500.

The planets are relatively close to the Earth and shine with light reflected from the Sun. The stars transmit their own light from an immense distance. Of the 4,850 stars visible to the naked eye, only the brightest concern the navigator, and they amount to twenty or so.

Stellar Magnitudes. It is customary to classify stars according to the amount of light received from them. That is, each star is given a magnitude or measure of its relative brightness.

This practice dates back to Hipparchus and Ptolemy, who arbitrarily graded the stars into six magnitudes. Stars of the sixth magnitude were those just visible to the naked eye.

The discovery by Sir John Herschel in 1830 that a first-magnitude star is about one hundred times brighter than a sixth-magnitude star caused the Ptolemaic grading to be modified slightly, and now stars are graded according to the definition that a first-magnitude star is one from which the Earth receives one hundred times as much light as it receives from a sixth-magnitude star.

On this definition a second-magnitude star is one hundred times brighter than a seventh-magnitude star; a third, one hundred times brighter than an eighth; and so on. Negative magnitudes are thus possible (and exist in fact) because the star which is one hundred times brighter than a fifth-magnitude star must be of magnitude 0; and a star which is that amount brighter than a fourth-magnitude star must be of magnitude -1.

The Sun's magnitude is $-26{\cdot}7$, and the Moon's (at the full) is $-12{\cdot}5$.

The intervening magnitudes between 1 and 6 are found from a logarithmic scale, so that, if a is the numerical index of the quantity of light received:

$$a^6 : a :: 100 : 1$$

i.e. $$a^5 = 100$$

$\therefore$ $$a = 2{\cdot}51$$

A first-magnitude star is therefore 2·51 times as bright as a second-magnitude star ; $(2{\cdot}51)^2$ times as bright as one of third magnitude ; $(2{\cdot}51)^3$ times as bright as one of fourth, and so on.

Vega, for example, with a magnitude of 0·1, is 2·51 times as bright as Aldebaran with a magnitude of 1·1 ; and Canopus, with a magnitude of —0·9, is two magnitudes brighter than Aldebaran. Sirius, the brightest star, has a magnitude of —1·6, and is 2·9 magnitudes brighter than Regulus (1·3). Sirius therefore gives $(2{\cdot}51)^{2{\cdot}9}$ or nearly 16 times the amount of light given by Regulus.

In practice the first-magnitude stars are those brighter than magnitude 1. They are, in order of brightness, Sirius, Canopus, α Centauri, Vega, Capella, Arcturus, Rigel, Procyon, Achernar, β Centauri, Altair and Betelgeuse.

The few navigational planets and stars have magnitudes of 2 and less. Stars brighter than magnitude 20 number about 1,000,000,000.

THE NAVIGATIONAL PLANETS

The declinations of the four navigational planets never exceed the limits of 26°N. and 26°S., and therefore an observer has a general idea about their positions in the sky. The mean distances of these planets and the Earth from the Sun are about :

Venus	—	67,000,000	miles
Earth	—	93,000,000	,,
Mars	—	142,000,000	,,
Jupiter	—	483,000,000	,,
Saturn	—	886,000,000	,,

Venus lies between the Earth and the Sun, and is therefore said to be an *inferior* planet. To an observer on the Earth it is never more than 47° removed from the Sun, for which reason it cannot be seen at night in these latitudes. It is thus a ' morning or evening planet '. Its magnitude varies slightly but is, on the average, —3·4. No other star or planet is so brilliant.

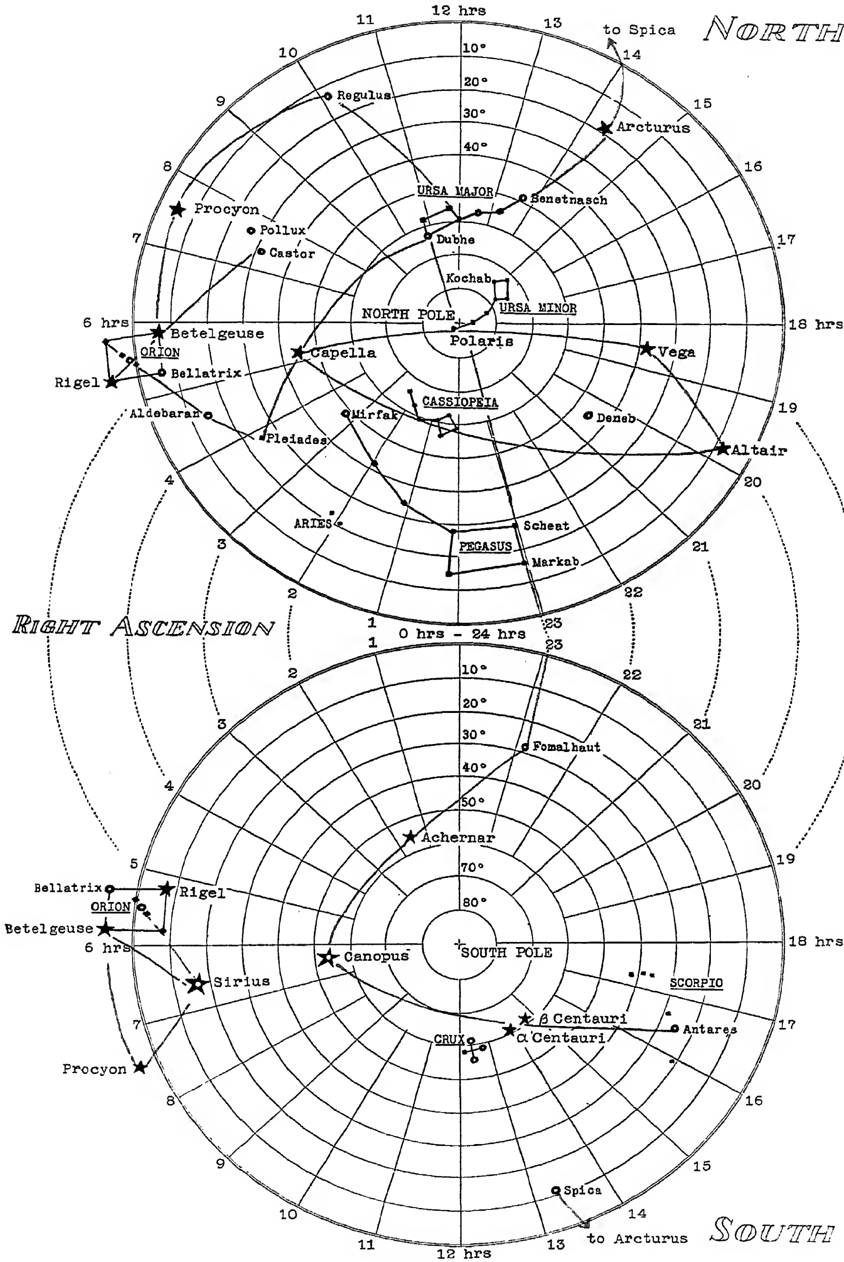
Mars, with an average magnitude of about —0·2, varies appreciably in brilliance, but is easily distinguished by its red light.

Jupiter has an average magnitude of —2·2 and ranks next to Venus in brilliance.

Saturn, with an average magnitude of 1·4, is not readily identified, and the methods of identification explained later in this chapter may have to be employed. Saturn's rings are not visible through the telescopes and binoculars normally used on the bridge.

THE CONSTELLATIONS

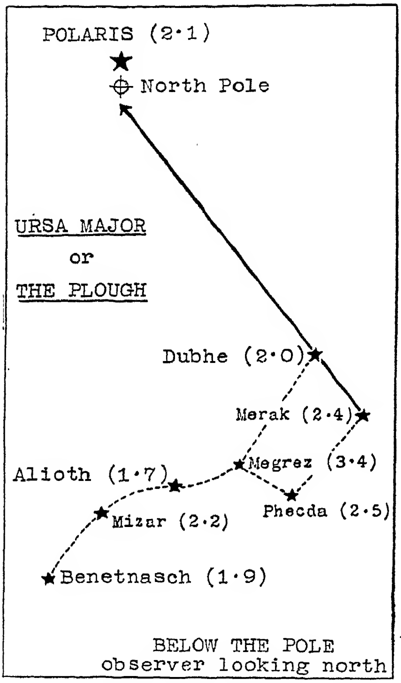
Of more general use to the navigator are the bright stars, because it is possible to identify them fairly quickly from the relative positions which they maintain. Mostly they fall within certain well-defined constellations, and once the observer is able to pick out the key constellations he should have no difficulty in picking out the stars.


NORTH
12 hrs
11
13
to Spica
14
10
15
9
16
8
17
7
6 hrs
18 hrs
19
20
4
21
3
22
2
1
23
10°
20°
30°
40°
Regulus
Arcturus
URSA MAJOR
Benetnasch
Procyon
Pollux
Castor
Dubhe
Kochab
URSA MINOR
NORTH POLE
Polaris
Betelgeuse
ORION
Capella
Vega
Rigel
Bellatrix
CASSIOPEIA
Aldebaran
Mirfak
Deneb
Pleiades
Altair
ARIES
Scheat
PEGASUS
Markab
RIGHT ASCENSION
0 hrs - 24 hrs
50°
70°
80°
Fomalhaut
Achernar
Bellatrix
Rigel
ORION
Betelgeuse
Canopus
SOUTH POLE
Sirius
SCORPIO
β Centauri
α Centauri
CRUX
Antares
Procyon
Spica
to Arcturus
SOUTH

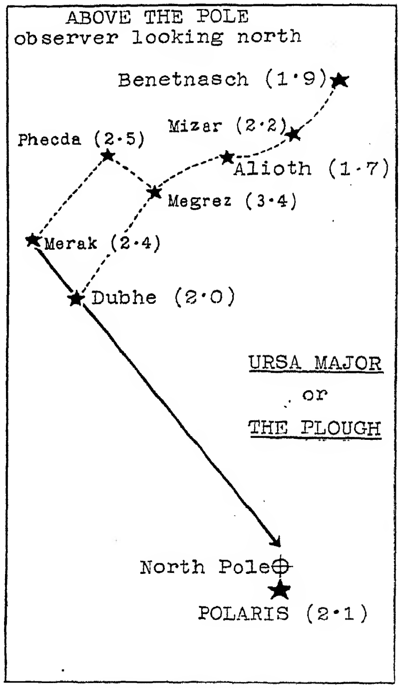
STAR CHART 1.

At the same time, it should be remembered that these constellations still carry the fanciful names bestowed upon them by the earlier astronomers. These names give but little assistance to the student of the night sky because the constellations seldom bear any resemblance to their classical descriptions.

Ursa Major or **The Great Bear.** This constellation is popularly known as *The Plough,* and it is important because a line drawn through its pointers carries the eye to the Pole Star. Chart 1 shows the position of *The Plough* in relation to the other constellations, but



Star Chart 2a.

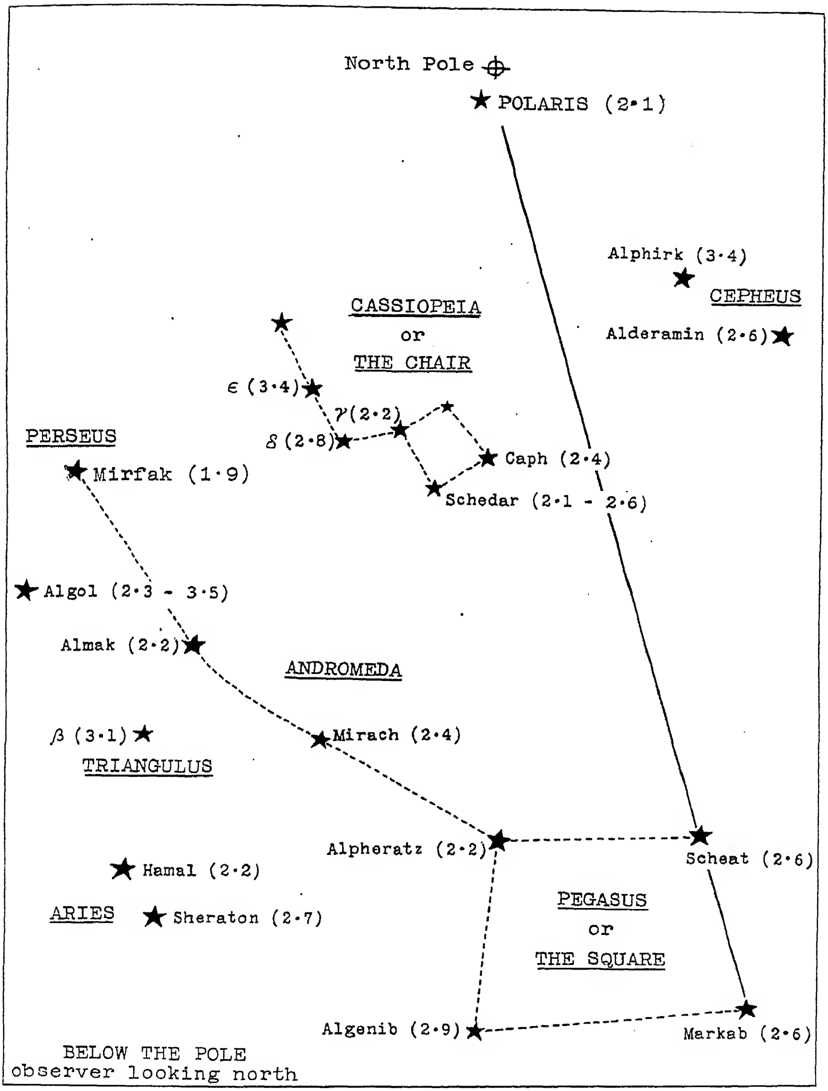


Star Chart 2b.

it must be borne in mind that this position in the visible sky is not fixed. In the latitude of England the entire constellation is circumpolar, and below the pole it will appear as shown in chart 2a, whereas above the pole it will be as in chart 2b.

These charts also indicate why the two stars Dubhe and Merak are referred to as ' the pointers '.

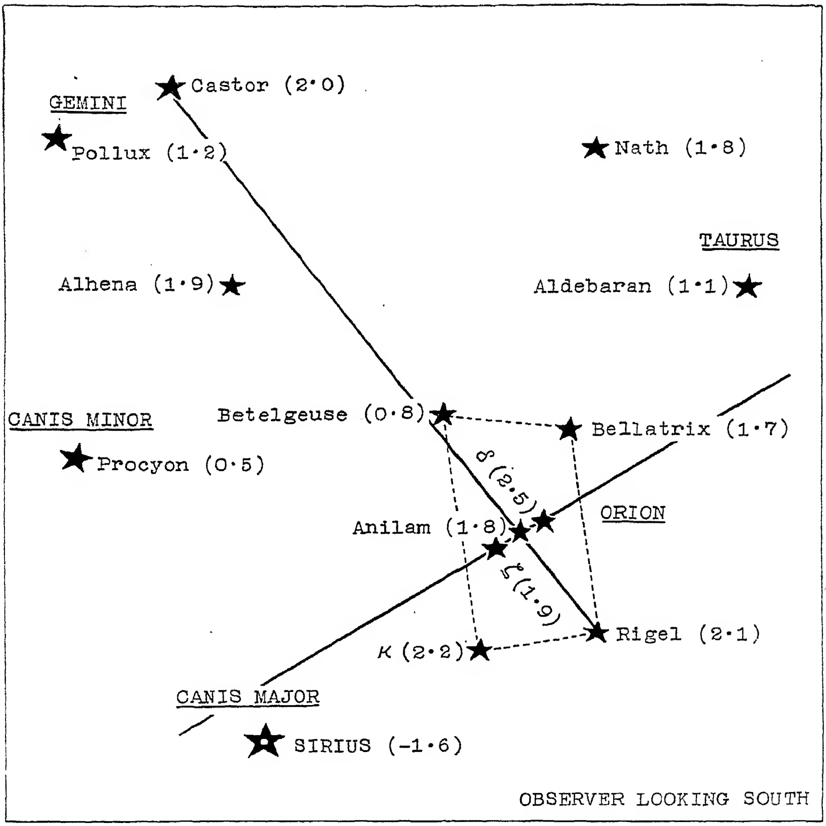
Ursa Minor or **The Little Bear.** This is not unlike *The Plough* in shape, but to the navigator its sole claim to distinction lies in its possession of the Pole Star, which marks the tip of the handle of the ' plough '.



STAR CHART 3.

Cassiopeia. This constellation—sometimes known as *The Chair*—is found on the side of the pole opposite to *Ursa Major* and about the same distance away. It does not contain any stars of first magnitude, but it is fairly prominent in the sky, and it is useful in helping the eye to pick up *Pegasus*. (Star chart 3.)

Pegasus. This constellation—sometimes known as *The Square*, although the figure that is formed by joining the four principal stars does not fulfil the Euclidean definition of one—is useful to anybody wishing to obtain some idea of sidereal time, because the side formed by Alpheratz and Algenib lies almost on the meridian through the First Point of Aries. (Star chart 3.)



STAR CHART 4.

Aries. The slowly accumulating result of the precession of the equinox, described in Chapter IX of Volume III, is seen in the distance of the constellation *Aries* from the meridian through the First Point of Aries.

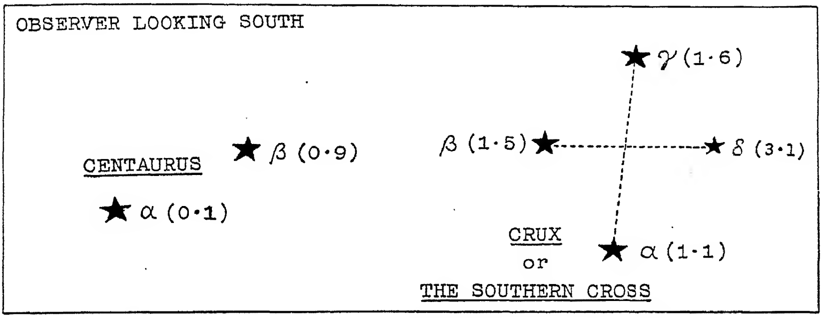
Orion. This important constellation contains stars of north and south declination. It is supposed to resemble a ' giant ', and the

three close stars in the centre of the constellation are referred to as *Orion's Belt.* (Star chart 4.)

The importance of this constellation lies in the signposts it affords the observer.

The 'belt' points almost directly at Sirius, 'the dog star', in the constellation of *Canis Major* or *The Great Dog*; and a line drawn through Rigel and its centre 'button' carries the eye straight to Castor in the constellation of *Gemini* or *The Twins.*

The constellations of *Canis Minor* or *The Little Dog* (which



STAR CHART 5.

contains Procyon) and *Taurus* or *The Bull* (which contains Aldebaran) lie close at hand, as shown in chart 4.

Crux or **The Southern Cross.** This constellation forms a cross only if the observer imagines diagonal lines joining the four stars in it. Its significance is more poetic than navigational, and it is too far removed from the south celestial pole to be of any use in finding the observer's latitude directly, as may be done with the Pole Star in the northern hemisphere. Two bright stars in the constellation *Centaurus*—star chart 5—help the observer to find it.

THE NAVIGATIONAL STARS

From his knowledge of the constellations just described, an observer should be able to pick out the navigational stars—if they are above the horizon—by referring them to imaginary lines in the celestial sphere.

Achernar (α *Eridani*, Mag: 0·6). This star lies midway between Canopus and Fomalhaut on the line joining them. (Chart 1.)

Aldebaran (α *Tauri*, Mag: 1·1). This star can be fixed in relation to Orion's Belt, which points roughly at it in one direction and at Sirius in the other and lies almost midway between them. (Chart 4.) Aldebaran is further distinguished by a reddish tint, and is at one extremity of a pronounced 'V'.

Altair (α *Aquilæ*, Mag: 0·9). A line from Capella through Caph in Cassiopeia points to Altair, which also lies between two less bright but prominent stars in a line with Vega. (Chart 1.)

Antares (α *Scorpii*, Mag: 1·2). This, another reddish star, lies at the centre of a small bow which points directly at another bow. (Chart 1.)

Arcturus (α *Bootis*, Mag: 0·2). This is one of the brightest stars, and is found by continuing the curve of the Great Bear's 'tail'. (Chart 1.)

Bellatrix (γ *Orionis*, Mag: 1·7). This is one of the three bright stars that mark the corners of the quadrilateral in the constellation of Orion. (Charts 1 and 4.)

Betelgeuse (α *Orionis*, Mag: 0·5–1·1). This is another of the three bright stars that mark the corners of the quadrilateral in the constellation of Orion. (Charts 1 and 4.)

Canopus (α *Argus*, Mag: —0·9). Next to Sirius, Canopus is the brightest star. It lies about half-way between Sirius and the south celestial pole, and on the line joining Fomalhaut and Achernar. (Chart 1.)

Capella (α *Aurigæ*, Mag: 0·2). This bright star lies on a line drawn from the Pole Star roughly at right-angles to the line joining 'the pointers', at a distance of about 45° from the pole on the side remote from the Little Bear. (Chart 1.) Near Capella are three fairly bright stars which form an isosceles triangle.

Castor (α *Geminorum*, Mag: 1·6). A line from Rigel through the middle star of Orion's Belt points to *Castor*. (Charts 1 and 4.)

α **Centauri** (Mag: 0·1) and β **Centauri** (Mag: 0·9). These are two bright stars on the line joining Antares and Canopus. (Chart 1.)

Fomalhaut (α *Piscis Australis*, Mag: 1·3). The line joining Scheat and Markab in Pegasus, produced away from the Pole Star, passes through Fomalhaut. (Chart 1.)

Polaris or **The Pole Star** (α *Ursæ Minoris*, Mag: 2·1). A line through 'the pointers' of the Great Bear leads to this star, and the observer can easily verify that he has chosen the correct star by measuring its altitude, which is roughly his latitude. (Charts 1 and 2.)

Pollux (β *Geminorum*, Mag: 1·2). This, as the name of the constellation suggests, will be seen close to Castor. (Charts 1 and 4.)

Procyon (α *Canis Minoris*, Mag: 0·5). Procyon forms, with Sirius and Betelgeuse, an equilateral triangle. (Charts 1 and 4.)

Regulus (α *Leonis*, Mag: 1·3). A line from Bellatrix through Betelgeuse points to Regulus, which is about 60° from Betelgeuse. (Chart 1.)

Rigel (*β Orionis*, Mag : 0·3). This is the third of the three bright stars that, with *κ* Orionis, form the quadrilateral in the constellation of Orion. (Charts 1 and 4.)

Sirius (*α Canis Majoris*, Mag : —1·6). Sirius is the brightest star. It lies to the south-east of Orion roughly in a line with the 'belt'. (Charts 1 and 4.)

Spica (*α Virginis*, Mag : 1·2). This bright star may be found by continuing the curve of the Great Bear's tail through Arcturus, which lies about midway between the tail and Spica. (Chart 1.)

Vega (*α Lyræ*, Mag : 0·1). Vega is found by extending the line joining Capella to the Pole Star about an equal distance on the opposite side of the pole. Near Vega is a distinct 'W' of small stars. (Chart 1.)

IDENTIFICATION OF STARS

In the practice of navigation, star sights are taken at morning and evening twilight, when the horizon and the few bright stars are visible at the same time. This means that there is usually no 'background' of constellations to assist the navigator in their identification. He must therefore find other methods.

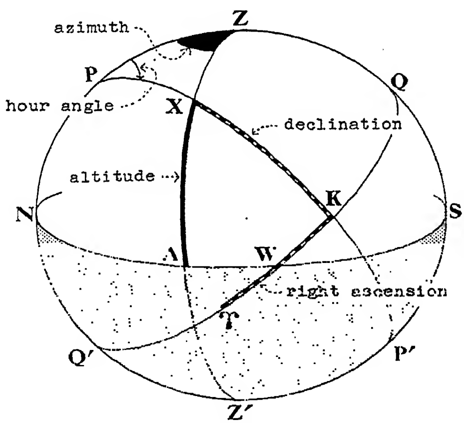


FIGURE 137.

A star is known when its right ascension and declination are known, and the observer's task is to find these quantities from the star's bearing and altitude, and the deck watch time.

That is, in figure 137, it is required to deduce ♈K, the right ascension, and KX, the declination, from AX, the altitude, the angle

PZX, the azimuth, and the angle ZPX, which is the hour angle found from the deck watch time.

Since $\Upsilon K = \Upsilon Q - KQ =$ R.A.M.—H.A.✱, the first step is to find the R.A.M.

The second is to solve the triangle PZX in which PZ (the co-latitude), ZX (90°—altitude) and the angle PZX (the azimuth) are known. This gives PX and therefore KX, the declination.

The Star Globe. This instrument gives a mechanical solution of the problem. It is easy to use and offers by far the best method of identifying a star in the given circumstances.

It consists, principally, of a globe on which the stars are shown in the places they occupy in the celestial sphere. Parallels of declination, the equator, the ecliptic and the meridians are also shown.

The globe revolves in a brass ring which is graduated in degrees and represents the meridian, and the whole fits into a hemispherical hollow in the box. The edge of the box is graduated in degrees of azimuth and represents the horizon. Over the globe, in the standard instrument, is fitted a hemispherical cage, the arms of which are graduated in degrees and correspond to circles of altitude. The observer's zenith lies at their point of intersection.

To use the star globe :

(1) Set the meridian ring to the latitude. This is most easily done by making the elevation of the pole equal to the latitude.

(2) Revolve the globe in the ring until the meridian marked with the calculated R.A.M. appears under the meridian ring.

(3) Turn the brass cage until one of the altitude circles lies along the observed true bearing of the body, and then move the small pointer along this semicircle to the required altitude.

The pointer will now indicate the star that has been observed, or, if no star is shown, the position of a planet.

What Star Is It? A small book entitled *What Star Is It?* (Harvey) gives solutions of the triangle PZX for the arguments of altitude and azimuth.

On the right-hand pages are tabulated the declinations for every 5° of latitude from 0° to 65°, the arguments being every 5° of altitude, along the top of each page, and every 10° of azimuth, down the side.

On the left-hand pages are tabulated the hour angles for a star in the western half of the celestial sphere, and if T is this tabulated hour angle, the star's right ascension is (R.A.M.—T). But the actual hour angle of a star in the eastern half is (24^h—T) and the star's right ascension is :

$$\text{R.A.M.} - (24^h - \text{T})$$
$$= \text{R.A.M.} + \text{T}.$$

Hence the rule :

Star East—add the tabulated hour angle to sidereal time in order to obtain the star's right ascension.

Star West—subtract the tabulated hour angle from sidereal time in order to obtain the star's right ascension.

At Z.T. 0455(+9) *on the* 3*rd April* 1937, *in D.R.* 43°18′*N.*, 140°03′*W.*, *the deck watch showed* $1^h54^m22^s$ *when the sextant altitude of a bright star was* 43°12′·2. *The deck watch was* 4^s *fast on G.M.T.*, *and the bearing of the star was* 125°.

Z.T.	0455 3rd April	
Zone	+9	
G.D.	1355 3rd April	
	h m s	
D.W.T.	1 54 22	
Error fast	4	
G.M.T.	13 54 —	3rd April
Long. W.	9 20	
L.M.T.	4 34	
R	12 46	
R.A.M.	17 20	

The star bears 125° or S.55°E. (true).

From *What Star Is It?* for an altitude of 43°, a true bearing of S.55°E., and latitude 43°N. :

declination=9°10′N. (pages 25 and 27)

	h m	
'T'	= 2 30	(pages 24 and 26)
R.A.M.	=17 20	
R.A.	=19 50	(added since the star is east)

In the list of stars in the *Nautical Almanac*, the only bright star that has a right ascension of approximately 19^h50^m and a declination of 9°10′N. is Altair, with a right ascension of $19^h47^m44^s$ and a declination of 8°42′N.

The Star Diagram. It will be seen from the above example that extreme accuracy is not necessary in finding the right ascension and

declination because if it should happen that several stars are close together, the one observed will certainly be the one of greatest magnitude. An ordinary diagram on the plane of the horizon may therefore be used to give the required arguments.

(1) *At Z.T.* 2200(+3) *on the* 2*nd April* 1937, *in* 50°*N.*, 44°*W.*, *the altitude of a bright star bearing* 105° *was* 40°. *What was the star?*

Z.T.	2200 2nd April
Zone	+3
G.D.	0100 3rd April

	h m	
Approximate G.M.T.	1 00	3rd April
Long. W.	2 56	
L.M.T.	22 04	
R	12 44	
R.A.M.	10 48	

In figure 138, *WNES* is the horizon and *Z* the observer's zenith. *Q* is placed so that, if *ZS* is 90 units, *ZQ* is 50 units, corresponding to the latitude of 50°N. *PZ* is then 40 units.

A circle drawn through *WQE* is taken as the celestial equator and divided into 24 spaces, starting from *Q* on the observer's meridian.

Between *Q* and *W*, the six spaces are equal, but between *W* and *Q'* the next six are stretched as shown because the arc *WQ'E* is really below the plane of the horizon and in the figure it is pulled out in order to show the position of ♈.

♈ is marked so that its hour angle, measured westward from the meridian, is $10^h 48^m$.

From *Z* the star's bearing *ZA* is drawn, the angle *PZA* being 105°.

X is placed so that *XA* is 40 units (corresponding to the altitude of *X*) and *ZX* is 50, making 90 units for *AZ*.

PX is drawn as a curved line and produced to cut the celestial equator at right-angles at *K*.

Since *KP* is 90°, *KX*, the declination, is about 20°N.; and ♈*K*, the right ascension, measured anti-clockwise, is about $14^h 15^m$.

The *Nautical Almanac* shows that the star was Arcturus, with a declination of 19°30′·3N., and a right ascension of $14^h 12^m 50^s$.

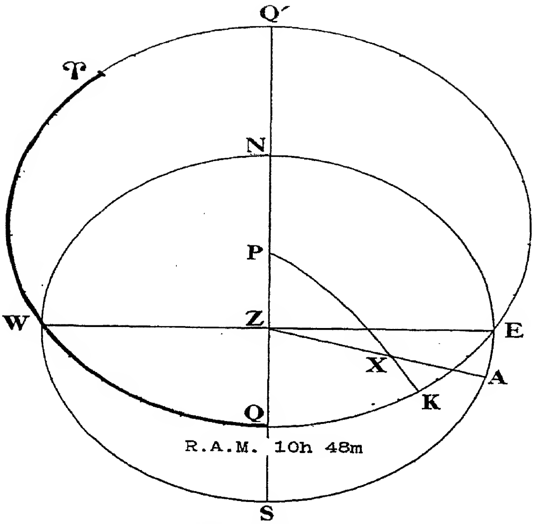


FIGURE 138.

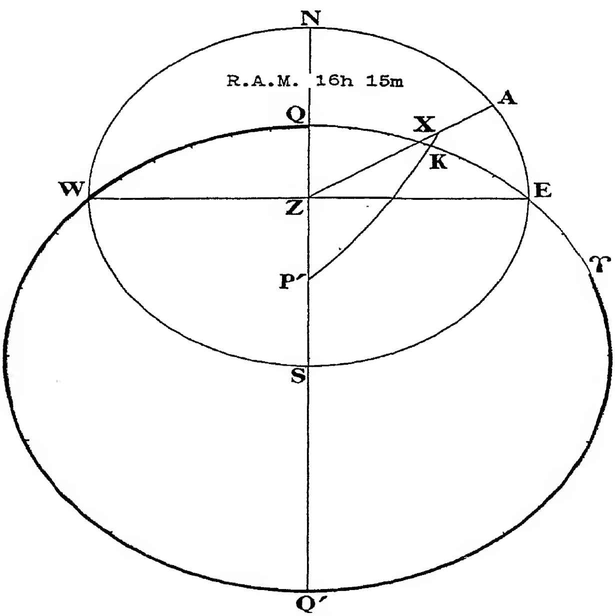


FIGURE 139.

(2) *At Z.T.* 0315(−1) *on the 3rd April* 1937, *in* 38°*S.*, 19°*E.*, *the altitude of a bright star bearing* 057° *was* 23°. *What was the star?*

Z.T.	0315 3rd April	
Zone	−1	
G.D.	0215 3rd April	

	h	m	
Approximate G.M.T.	2	15	3rd April
Long. E.	1	16	
L.M.T.	3	31	
R	12	44	
R.A.M.	16	15	

The star diagram (figure 139) shows that the star's declination is about 10°N., and its right ascension about 19^h30^m. The *Nautical Almanac* gives the star as Altair.

If necessary, a star diagram can be used for finding stars suitable for observation at a particular moment. The diagram is then drawn for the R.A.M. at the moment in question; the stars are plotted and a selection made.

CHAPTER XXII

THE RISING AND SETTING OF HEAVENLY BODIES

A knowledge of the rising and setting of heavenly bodies is essential to the navigator because, for one reason, the times at which he can take his star sights are governed by the times of sunrise and sunset.

Theoretical Rising and Setting. Theoretical rising or setting occurs when the centre of the heavenly body is on the observer's celestial horizon, east or west of his meridian. At these times the true zenith distance is 90°.

Visible Rising and Setting. These phenomena occur when the upper limb of the heavenly body is just appearing above or disappearing below the observer's visible horizon. It will be shown later in this chapter that only the Moon's centre lies practically on the celestial horizon at these moments, and that when the Sun's centre lies there, the Sun itself is appreciably above the visible horizon.

SUNRISE AND SUNSET

Visible Sunrise and Sunset. Visible sunrise or sunset occurs when the Sun's upper limb appears on the visible horizon. At this moment the observed altitude of the Sun's upper limb is 0°0′.

True Altitude at Visible Sunrise and Sunset. By correcting this zero altitude, the true altitude and therefore the true zenith distance of the Sun's centre can be found. Thus, if the observer is assumed to have no height of eye, and the Sun's semi-diameter on the day in question is 16′·0 :

Obs. Alt.	0°00′·0
Refraction	—35′·4
	—0°35′·4
Semi-diameter	—16′·0
True altitude	—0°51′·4

The true zenith distance is therefore 90°51′·4, and the Sun's centre is about 1° below the celestial horizon when its upper limb

is just visible. For this reason visible sunrise occurs *before* theoretical sunrise, and visible sunset *after* theoretical sunset.

Apparent Altitude at Theoretical Sunrise and Sunset. At the instant of theoretical sunrise or sunset, the true altitude of the Sun's centre is 0°0′, and the approximate observed altitude of the Sun's lower limb can be deduced thus, the Sun's semi-diameter again being taken as 16′·0 :

True Alt.	0°00′·0
Semi-diameter	—16′·0
	—0°16′·0
Refraction	+32′·0
Obs. Alt.	0°16′·0

At theoretical sunset, therefore, the Sun's lower limb appears to be a semi-diameter above the visible horizon.

The refraction in this example corresponds to an observed altitude of 0°16′, and thus differs from the refraction (35′·4) in the previous example, which corresponds to one of 0°0′.

Times of Theoretical Sunrise and Sunset. These times are not usually required in the practice of navigation, but if they should be, they can be found by solving the spherical triangle for the angle at the pole when the zenith distance is 90°. This angle is the H.A.T.S. of sunrise or sunset. Some of the earlier patterns of altitude-azimuth tables also give the apparent times of rising and setting at the bottom of each column.

Times of Visible Sunrise and Sunset. The *Nautical Almanac* includes tables for visible sunrise and sunset. These tables solve the triangle *PZX* for the angle at *P* when the true zenith distance of the Sun's centre is 90°50′.

Northern Latitudes

The tables are constructed for northern latitudes, and they give the exact L.M.T. of the phenomenon on the Greenwich meridian. This time would also be the exact L.M.T. on any other meridian, if the same latitude and declination were used for solving the triangle *PZX*. But the Sun's declination changes during the course of the day, with the result that there is a small 'daily difference' of a minute or two in the times of successive phenomena. The time given in the tables is therefore an approximate L.M.T. of the phenomenon on any meridian other than that of Greenwich. This small daily variation has, however, no practical significance, and it is customary to regard the time given in the tables as the L.M.T. on any meridian.

The zone time of sunset in 52°N., 33°W. on the 3rd April 1937, for example, would be found thus :

	h m	
L.M.T. of sunset at 52°N., 33°W.	18 36	3rd April
Longitude (33°W.)	2 12	
G.M.T. of sunset at 52°N., 33°W.	20 48	3rd April
Zone (+2)	—2	
Z.T.	1848(+2)	3rd April

Southern Latitudes

In order to find the times of sunrise and sunset in southern latitudes, it is assumed that the times of these phenomena at 45°S., say, are the same as those at 45°N. on a date approximately six months earlier or later, because the Sun's declination is then the same in actual amount but opposite in name. A situation is therefore produced in the northern hemisphere identical with that in the southern.

An auxiliary table gives the date on which the northern tables must be entered, together with a small correction which is the difference in the equations of time on the two days.

If, for example, the time of sunset in 45°S., 112°W. on the 3rd April 1937 is required, the main tables must be entered for latitude 45°N. on the 7th October, and a correction of 16^m added to the time thus found to give the L.M.T. at the southern place. The longitude can then be applied and zone time found as before. Thus :

	h m	
L.M.T. of sunset at 45°N., 112°W.	17 31	7th October
Correction	+16	
L.M.T. of sunset at 45°S., 112°W.	17 47	3rd April
Longitude (112°W.)	7 28	
G.M.T. of sunset at 45°S., 112°W.	1 15	4th April
Zone (+7)	—7	
∴ Z.T.	1815(+7)	3rd April

TWILIGHT

Morning and Evening Twilight. With the tables giving sunrise and sunset are smaller tables giving the beginning of morning twilight and the end of evening twilight. These twilight tables are used in exactly the same way as the sunrise and sunset tables.

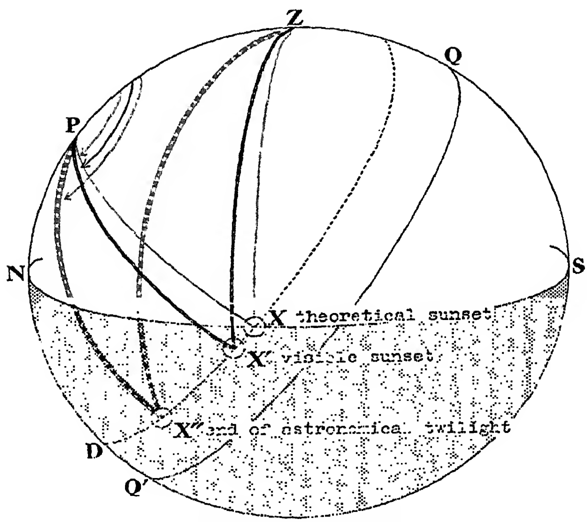
Twilight is that period of the day when, although the Sun is below the horizon, the observer is still receiving light reflected and

scattered by the upper atmosphere. For convenience this period is divided into three stages :

(1) *Civil Twilight.* This is said to end or begin when the Sun's centre is 6° below the horizon, and it is roughly the time when the horizon begins to grow indistinct or become clear.

(2) *Nautical Twilight.* This is said to end or begin when the Sun's centre is 12° below the horizon.

(3) *Astronomical Twilight.* This is said to end or begin when the Sun's centre is 18° below the horizon, at which moment absolute darkness is assumed to begin or end so far as the Sun is concerned.



POSITION OF THE SUN

FIGURE 140.

Duration of Twilight. Morning twilight, whether civil, nautical or astronomical, begins when the Sun's centre is at the appropriate depression below the horizon, and lasts until visible sunrise.

Evening twilight begins at visible sunset and lasts until these depressions are reached.

Figure 140 shows the relative positions of the Sun at theoretical and visible sunset, and at the end of astronomical twilight—X, X' and X'' being the positions ; ZPX, ZPX' and ZPX'' the corresponding hour angles ; and ZX(90°), ZX'(90°50′) and ZX''(108°) the zenith distances.

If the circle of declination does not fall 18° below the horizon, then astronomical twilight does not end. This occurs when the latitude and declination have the same name and their sum is not less than 72°.

In figure 140, PN is the latitude, DQ' the declination ; and PQ' is 90°. Therefore, if $(PN+DQ')$ is greater than 72°, DN must be less than 18°.

Similar limits for civil and nautical twilight are obtained by writing 6° and 12° instead of 18°.

'The Midnight Sun.' If the circle of declination does not reach the horizon, then the Sun can never set. In the northern hemisphere, the limiting latitude for this to occur is, from figure 141, (90°—the Sun's greatest northerly declination) ; that is $(90°-23\frac{1}{2}°)$ or $66\frac{1}{2}°$. This limit is reached on one day only during the year.

For a similar reason, astronomical twilight will last all night on one night of the year in $(90°-23\frac{1}{2}°-18°)$N. ; that is, in $48\frac{1}{2}°$N.

These limits, with their names altered to south, also apply in southern latitudes.

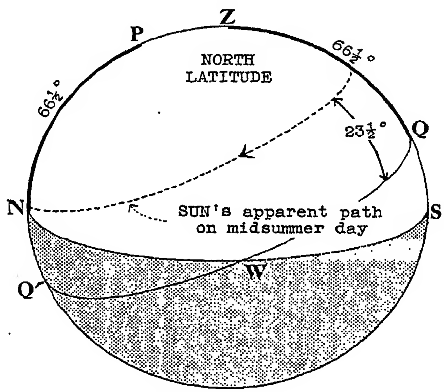


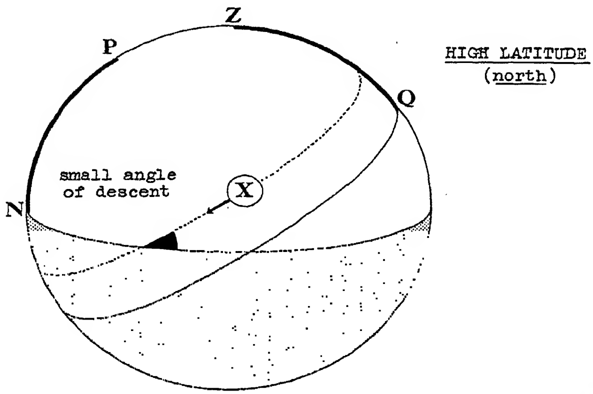
FIGURE 141.

Since astronomical twilight lasts until the Sun's centre is 18° below the horizon, the actual duration of twilight depends on the angle which the Sun's path makes with the horizon. If that angle is small, as it must be in high latitudes (figure 142a), twilight lasts considerably longer than it does in low latitudes where the angle is large (figure 142b). Thus twilight in the tropics usually lasts about an hour, but in the south of England, at midsummer, for example, it lasts all night.

Artificial light becomes necessary when the Sun is about 6° below the horizon.

The duration of any form of twilight is obtained from the tables, the difference being taken between the time of visible sunset and the end of twilight, or the beginning of twilight and sunrise ; but it can be calculated, if necessary, because it is simply the angle $X'PX''$ (in figure 140) and that is the difference between the hour angles in the two triangles PZX' and PZX''.

Circumpolar Bodies. Although every heavenly body is circumpolar in that, to an observer on the Earth, it describes a circle about the pole, the term *circumpolar*, when prefixed to a heavenly body, denotes that the heavenly body never sets and is always above the observer's horizon.



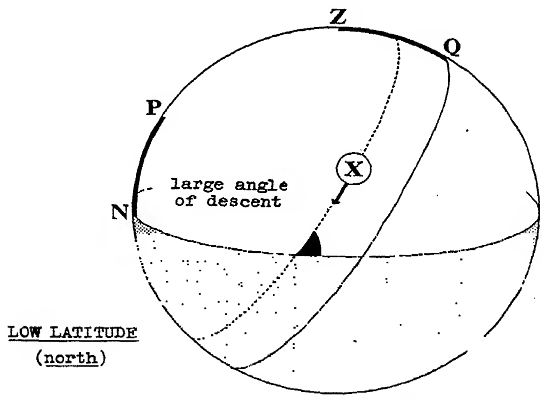


FIGURE 142a. FIGURE 142b.

MOONRISE AND MOONSET

The times of moonrise and moonset may be found by solving the triangle PZX for the hour angle in the same way that it can be solved for the Sun, but the calculation is even more laborious because the Moon's declination and right ascension are both changing so rapidly that a method of successive approximation must be employed in

order to obtain the proper declination and right ascension at the instant of moonrise or moonset.

To avoid this, tables are incorporated in the *Nautical Almanac* giving the times, to an observer on the Greenwich meridian with no height of eye, when the Moon's upper limb is just touching the visible horizon.

At this instant the true altitude of the Moon's centre is approximately :

Obs. Alt.	0°00′
Refraction	—35′
	—0°35′
Semi-diameter	—16′
Parallax (average)	+57′
True altitude	0°06′

Therefore, when the Moon's upper limb is touching the visible horizon, the Moon's centre is roughly on the celestial horizon.

The Moonrise and Moonset Tables. Like the tables for sunrise and sunset, these tables are constructed for northern latitudes and give the exact L.M.T. of the phenomenon on the Greenwich meridian. This time is also the approximate L.M.T. of moonrise or moonset at any other place having the same latitude. But, whereas the L.M.T. of sunrise or sunset changes only by a minute or two from one day to the next, the L.M.T. of moonrise or moonset may change by over an hour. For moonrise and moonset, in general, this approximate L.M.T. is correct only within about half an hour, and to make it exact, a proportion of the daily difference must be applied in the same way that it is applied when the time of the Moon's meridian passage is found. (Page 172, Chapter XIV.)

The proportion applied is :

$$\frac{\text{longitude in degrees}}{360^\circ} \times (\text{daily difference})$$

When the observer is in *west* longitude, this proportion is *added* to the L.M.T. of the phenomenon at Greenwich in order to give the L.M.T. on the observer's meridian, and the difference is taken between the day in question and the next following.

When the observer is in *east* longitude, this proportion is *subtracted* from the L.M.T. of the phenomenon at Greenwich in order to give the L.M.T. on the observer's meridian, and the difference is taken between the day in question and the preceding day.

The following example shows the procedure :

What is the time of moonrise in 50°N., 33°E., on the 3rd April 1937 ?

	h m	
L.M.T. of moonrise at 50°N., 0°E.	1 16	3rd April
Proportion (46×33÷360)	—4	
L.M.T. of moonrise at 50°N., 33°E.	1 12	3rd April
Longitude (33°E.)	2 12	
G.M.T. of moonrise at 50°N., 33°E.	23 00	2nd April
Zone (—2)	+2	
∴ Z.T.	0100(—2)	3rd April

Moonrise and Moonset in Southern Latitudes. Since the Moon's centre is approximately on the celestial horizon when the Moon's upper limb is on the visible horizon, the tables giving moonrise and

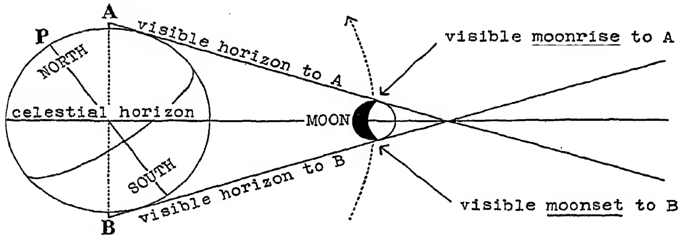


FIGURE 143.

moonset at a northern station may be used for giving the opposite phenomena at the antipodal point. This point has a latitude equal in value but opposite in name to that of the southern station, and differs in longitude by 180° or 12^h. The times of moonrise and moonset at a southern station can therefore be deduced from the times of moonset and moonrise at the corresponding northern station.

This method, of which figure 143 is a diagrammatic representation, assumes that the zenith distance at moonrise or moonset is exactly 90°, and a small correction, for which tables are given in the standard edition of the *Nautical Almanac,* is strictly necessary ; but this correction is usually ignored in practice.

Since the tables are entered for a station differing by 12^h from the actual station, the introduction of this 12^h will mean that the time of the phenomenon is found on the wrong date in certain circumstances unless the tables are entered for the previous or following day according to these circumstances.

If the southern station is east, and the time of the phenomenon at the northern station on the day in question is found to lie between

12^h and 24^h, the tables must be entered for the previous day. If the southern station is west, and the time of the phenomenon at the northern station lies between 0^h and 12^h, the tables must be entered for the following day. Thus:

(1) *What is the zone time of moonrise in* 35°*S.*, 128°*W.*, *on the 3rd April* 1937?

The northern station is 35°N., 52°E.

	h m	
L.M.T. of *moonset* at 35°N., 0°E.	11 28	*4th April*
Proportion ($56 \times 52 \div 360$)	−8	
L.M.T. of moonset at 35°N., 52°E.	11 20	4th April
Longitude (52°E.)	3 28	
G.M.T. of moonset at 35°N., 52°E.	7 52	4th April
∴ G.M.T. of *moonrise* at 35°S., 128°W.	7 52	4th April
Zone (+9)	−9	
∴ Z.T.	2252(+9)	*3rd April*

(2) *What is the zone time of moonset in* 20°*S.*, 76°*E.*, *on the 3rd April* 1937?

The northern station is 20°N., 104°W.

It is seen from the tables that the Moon does not rise at this station on the actual day, 3rd April. It rises at 23^h50^m on the 2nd.

	h m	
L.M.T. of *moonrise* at 20°N., 0°W.	23 50	*2nd April*
Proportion ($46 \times 104 \div 360$)	+13	
L.M.T. of moonrise at 20°N., 104°W.	0 03	3rd April
Longitude (104°W.)	6 56	
G.M.T. of moonrise at 20°N., 104°W.	6 59	3rd April
∴ G.M.T. of *moonset* at 20°S., 76°E.	6 59	3rd April
Zone (−5)	+5	
∴ Z.T.	1159(−5)	*3rd April*

In the 1939 and subsequent editions of both the abridged and the standard *Nautical Almanacs*, complete tables of moonrise and moonset will be included, and the method illustrated by the above examples will no longer be necessary.

CHAPTER XXIII

THE PRINCIPLE OF THE SEXTANT

The optical principle on which a sextant is constructed is that, if a ray of light is reflected from two mirrors in succession, the angle between the first and last directions of the ray is twice the angle between the mirrors.

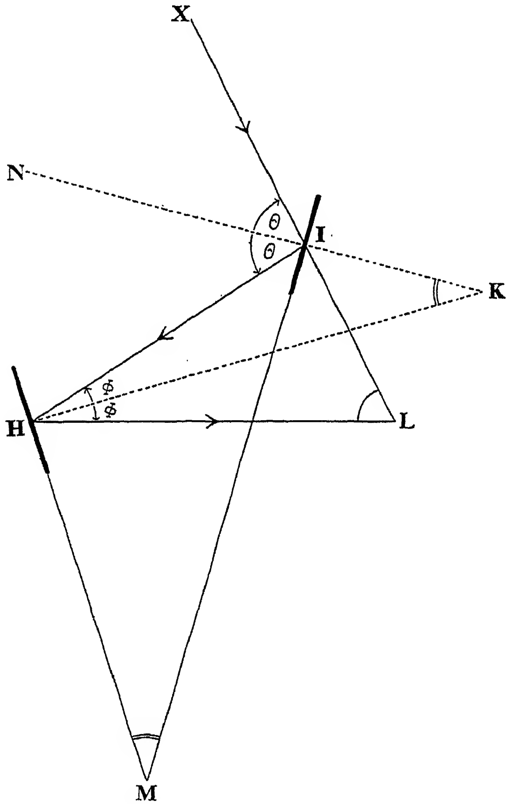


FIGURE 144.

In figure 144, *XIHL* is the path of a ray which is reflected from two mirrors; *I*, the index glass, and *H*, the horizon glass. *XI* meets *HL* in *L*. The angle *HLX* is therefore the angle between the first and last directions of the ray.

The angle between the mirrors is HMI, and this is equal to HKI, the angle between the normals at I and H.

The angles between the ray and these two normals are θ and ϕ.

Since an exterior angle of a triangle is equal to the sum of the internal and opposite angles, it follows that:

$$\angle HKI = \theta - \phi \quad (\text{in } \triangle HKI)$$
$$\angle HLI = 2\theta - 2\phi \quad (\text{in } \triangle HLI)$$

The angle between the first and last directions of the ray is therefore twice the angle between the mirrors.

Figure 63 on page 78 shows a pointer attached to the mirror I and moving over a scale of angles. This scale is marked as if the angles were twice their actual size in order to embody the principle that the angle between the first and last directions of the ray is

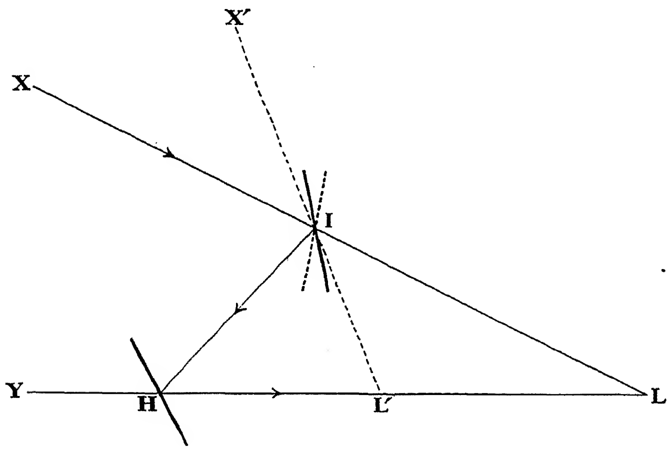


FIGURE 145.

twice the angle between the mirrors. The scale itself is, in length, about one-sixth of the circumference of a circle. Hence the name *sextant*.

In the actual sextant, the horizon glass is silvered for only half its height, and for that half it acts as a mirror, reflecting the ray that reaches it from the index glass, which is wholly a mirror. Through the unsilvered part, rays pass without hindrance and objects are seen directly.

In figure 145, Y is such an object, the ray from which coincides with the part HL of the reflected ray from X, and the angle between the first and last rays is XLY.

If another object X' is taken and the reflected ray from this made to coincide with YH produced by rotating the index glass to the dotted position shown, the angle between the first and last rays

is seen to be $X'L'Y$. The point on YH produced at which the angle between two objects is subtended therefore changes its position with the angle.

The actual positions of L and L', however, are unimportant because the angle read on the sextant is, by the construction of the sextant, twice the angle between the mirrors, and that is always equal to an angle of which XLY is typical. The size of the angle between the mirrors is, moreover, determined by the only movable mirror, that is I, the index glass, and it is at the index glass, therefore, that the angle subtended by X and Y must be measured. It is thus the angle XIY that is always wanted, and the distance of an object from the sextant, or, more accurately, from the point on the axis about which the index glass rotates, becomes in consequence a factor which cannot be ignored.

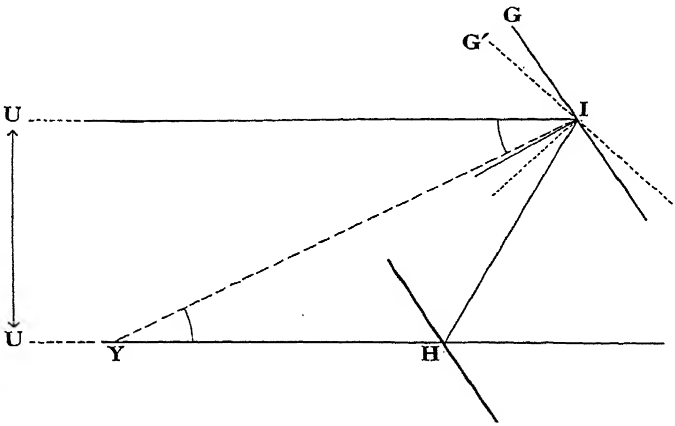


FIGURE 146.

Sextant Parallax. The angle subtended at an object by the index glass and the horizon glass of a sextant is known as the *sextant parallax*.

In figure 146, U is an object so distant that the lines UI and UH are parallel. When the index glass is parallel to the horizon glass, the direct and reflected images of U therefore coincide. But when the object is at Y, close to the sextant, the index glass must be rotated through a small angle GIG' before coincidence occurs, this small angle being equal to half the angle IYH, that is to half the sextant parallax. If, for the moment, it is assumed that there is no index error, the angle read on the sextant arc is thus the sextant parallax.

When the sextant is used for measuring the angle subtended by two separate objects X and Y—see figure 147—instead of the angle between the direct and reflected images of the same object, the angle actually measured is XLY whereas the angle required is XIY.

This angle, XIY, being an exterior angle of the triangle ILY, is equal to the sextant parallax plus the angle at L. That is:

$$\angle XIY = 2 \text{ (angle between the mirrors)} + p$$
$$= \text{angle measured on the sextant arc} + p.$$

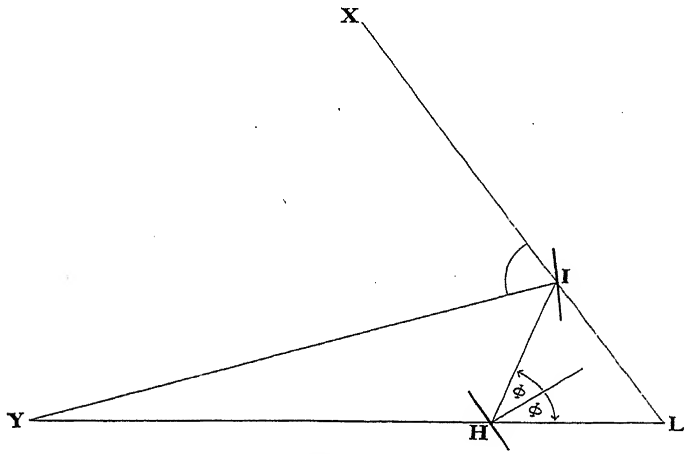


FIGURE 147.

The value of p can be found by applying the rule of sines (Appendix, page 242) to the triangle IHY. Thus:

$$\frac{IH}{\sin p} = \frac{IY}{\sin IHY}$$

But the angle IHY is equal to $(180° - 2\phi)$. Therefore:

$$\sin p = \frac{IH \sin 2\phi}{IY}$$

This formula shows that the sextant parallax decreases as the distance of Y increases. If, for example, ϕ is $17\frac{1}{2}°$ and IH is 3 inches, corresponding values of p and the distance of Y are:

p	IY
1″	5·33 miles
10″	4·6 cables
60″	156 yards

Sextant parallax is therefore negligible unless Y, the object viewed directly in the telescope, is close to the observer. For this reason, when the angle between two objects is measured, the telescope should always be pointed at the more distant object.

When it is necessary to measure the angle subtended by objects close at hand, the sextant parallax may be found by bringing the

direct and reflected images of the object Y into coincidence. The angle shown on the sextant is then a combination of the index error and the sextant parallax, and this angle may be applied as a correction to any angle measured between Y and another object, so long as the telescope is pointed at Y.

The Vernier. In order to increase the accuracy with which the graduated arc of the sextant may be read, a second and smaller graduated arc is read in conjunction with it. This smaller arc is called a *vernier*.

It is, for example, required to make a vernier for an ordinary foot rule graduated in inches, so that the rule can be read to one-tenth of an inch.

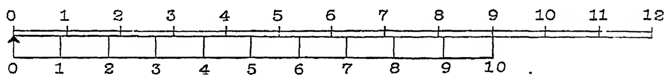


FIGURE 148.

A vernier that would enable this to be done would be a scale 9 inches long divided into 10 equal parts as shown in figure 148.

It will be seen that when the zero on the vernier is put under the zero on the rule, the first vernier division lies one-tenth of an inch to the left of the first division on the rule; the second vernier division lies two-tenths of an inch to the left of the second division

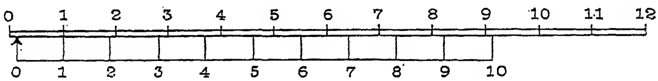


FIGURE 149.

on the rule; and so on until the last vernier division coincides with the ninth division on the rule. The point at which the rule is read is the point opposite the zero on the vernier, and as the vernier is now placed, the reading on the rule is zero.

Figure 149 shows the vernier moved so that the first division on it is opposite the first division on the rule. The distance it has

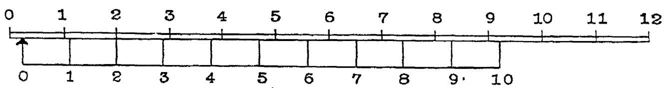


FIGURE 150.

been moved is clearly one-tenth of an inch, and the zero on the vernier therefore lies one-tenth of an inch to the *right* of the zero on the rule. The reading of the rule is thus 0·1 inches, corresponding to coincidence at the *first* vernier division.

Figure 150 shows the vernier moved so that the second vernier division is opposite the second division on the rule. The zero on

the vernier therefore lies two-tenths of an inch to the right of the zero on the rule. The reading of the rule is thus 0·2 inches, corresponding to coincidence at the *second* vernier division.

Figure 151 shows the vernier moved so that the second vernier division is opposite the third division on the rule. That is, the

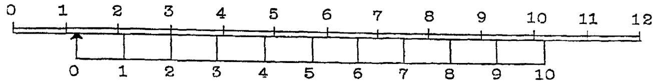


FIGURE 151.

vernier has been moved 1 inch further to the right. The zero on the vernier scale therefore lies two-tenths of an inch to the right of the first division on the rule. The reading of the rule is thus 1 inch on the rule and 0·2 on the vernier, making 1·2 inches and again corresponding to coincidence at the *second* vernier division.

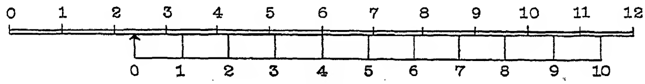


FIGURE 152.

Figure 152 shows coincidence at the *fourth* vernier division, and the rule reading 2·4 inches. Hence :

The number of the vernier division at which coincidence occurs is the number of the sub-division of the main unit.

If the rule were 2 feet instead of 1 foot, a vernier reading to one-tenth of an inch could be constructed with larger divisions by making 10 vernier divisions correspond to 19 arc divisions.

If it were necessary to make the vernier read to one-eighth of an inch instead of one-tenth, 8 vernier divisions would be made to correspond to 7 arc divisions, or to 15 or 23.

It should thus be clear that the number of vernier divisions must be equal to the same number of arc divisions less one, or to some multiple of that number less one. That is, in general, n vernier divisions are equal to $(n-1)$ or to $(mn-1)$ arc divisions ; and if the vernier division is v units in length and the arc division a units :

$$nv=(n-1)a \quad \text{or} \quad nv=(mn-1)a$$

But the scale can be read to a fraction of the main unit which is equal to the difference between an arc division and a vernier division. The least reading is therefore :

$$a-v \quad \text{or} \quad ma-v$$

By substitution :

$$a-\frac{(n-1)a}{n} \quad \text{or} \quad ma-\frac{(mn-1)a}{n}$$

i.e.

$$\frac{a}{n} \quad \text{or} \quad \frac{a}{n}$$

The accuracy of a vernier is therefore the size of one division of the scale divided by the number of divisions on the vernier, and if there is coincidence at the xth division of a vernier, the scale reading will be the number of scale divisions of the arc indicated by the zero of the vernier plus $(x \times a/n)$.

The vernier on the ordinary sextant used at sea is usually graduated so as to read to $0'{\cdot}2$.

This reading is achieved by making 50 vernier divisions correspond to 99 arc divisions. One arc division is $10'$, and the vernier therefore reads to one-fiftieth of $10'$, which is $12''$ or $0'{\cdot}2$.

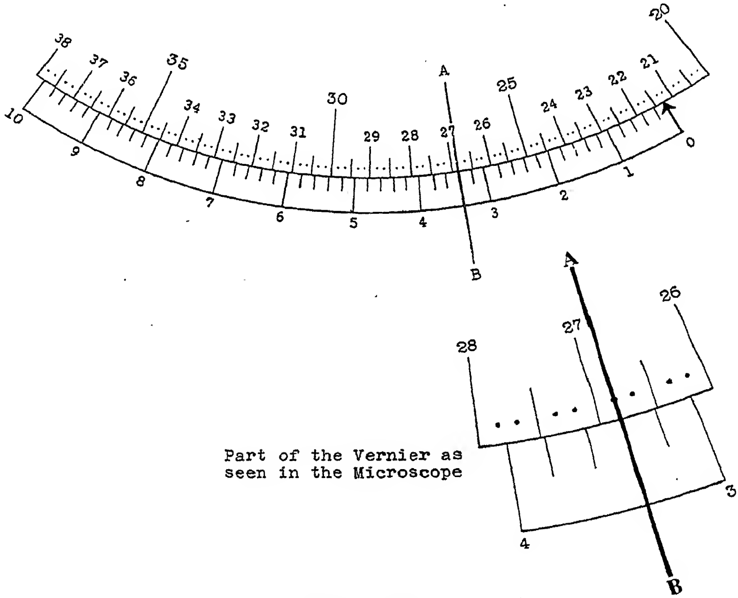


FIGURE 153.

Figure 153 shows a vernier thus constructed.

The scale reading is seen to be $21°10'$ (by reference to the zero of the vernier) plus a quantity which the vernier itself gives as $3'{\cdot}4$, because there is coincidence on the seventeenth vernier division. That is, the zero on the vernier lies to the left of the $21°10'$ division by an amount :

$$17 \times \frac{10}{50}$$

$$=3'{\cdot}4$$

The sextant reading is therefore $21°13'{\cdot}4$.

APPENDIX

Part I A Summary of Plane Trigonometry

Part II A Summary of Spherical Trigonometry

Part III Comparison of Methods of Working Sights.

Part IV Extracts from the *Nautical Almanac*

Part V Extracts from the *Air Almanac*

PART I

A SUMMARY OF PLANE TRIGONOMETRY

Trigonometry is the science of finding the angles and sides of a triangle when certain of these angles and sides are given. Plane trigonometry is the science applied to plane triangles. Spherical trigonometry is the science applied to the triangles marked on the surface of a sphere by planes through its centre.

The Degree. The angle between two lines is the inclination of one line to the other, and this inclination is commonly measured in degrees and sub-divisions of a degree.

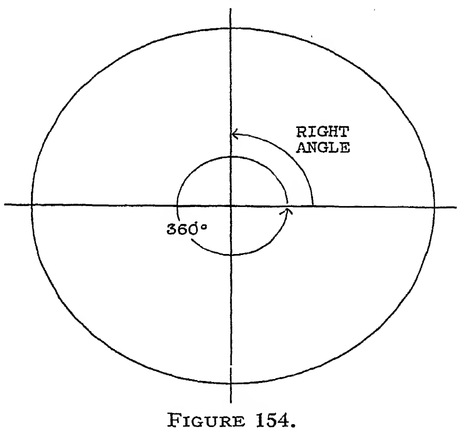


FIGURE 154.

In one complete revolution there are 360°. (Figure 154.) When the two arms of the angle are perpendicular, the angle is said to be a right angle, in which there are 90°.

The sub-divisions of the degree are the minute and the second, the connexion between them being :

$$1^\circ = 60'$$
$$1' = 60''$$

The Radian. The degree is an arbitrary unit. The principles of trigonometry would not be altered if its size were chosen so that one hundred degrees formed a right angle. The mathematical unit is the radian which is defined as the angle subtended at the centre of a circle by a length of arc equal to the radius.

The ratio of the circumference of a circle to its diameter is constant and equal to 3·14159 . . ., a number denoted by π. From this it follows that :

(1) the angle subtended by an arc equal to the radius is also constant and equal to $360°\div 2\pi$, or approximately $57°17'45''$.

(2) the number of radians in a right angle is $\frac{1}{2}\pi$.

(3) the length of any arc is equal to the radius multiplied by the angle in radians.

Trigonometrical Functions. There are six of these functions, but only two of fundamental importance, because the other four are derived from them. These two are known as the *sine* and *cosine* of the angle.

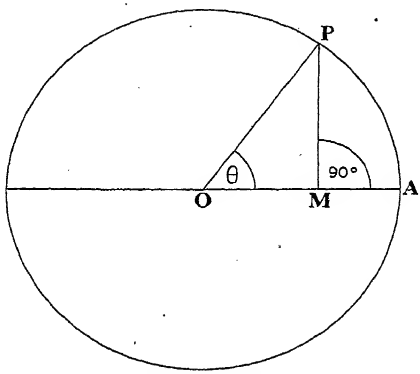


FIGURE 155.

In figure 155, POM is an angle θ where θ lies between 0° and 90°, and PM is the perpendicular from P on to the arm OA, so that OP, MP and OM form a triangle right-angled at M.

The six trigonometrical functions are the sine and cosine, tangent and cotangent, secant and cosecant ; and they are defined and abbreviated thus :

$$\sin\theta=\frac{\text{side opposite the angle}}{\text{hypotenuse}}=\frac{MP}{OP}$$

$$\cos\theta=\frac{\text{side adjacent to the angle}}{\text{hypotenuse}}=\frac{OM}{OP}$$

$$\tan\theta=\frac{\text{side opposite}}{\text{side adjacent}}\qquad MP$$

$$\cot\theta=\frac{OM}{MP}=\frac{}{\tan\theta}$$

$$\sec\theta=\frac{OP}{OM}=\frac{1}{\cos\theta}$$

$$\theta=\frac{OP}{MP}=\frac{}{\sin\theta}$$

The last three functions are thus reciprocals of the first three, and :

$$=\frac{MP}{OM}=\frac{MP}{OP} \wedge \frac{}{OM}=\frac{}{\cos\theta}$$

The Sign of the Six Functions. The sign of the six trigonometrical functions just defined varies with the size of the angle.

In figure 156, XOX' and YOY' are two straight lines cutting at right-angles in O and forming two axes along which all lengths are measured. The convention is that along OX or OY—to the right or towards the top of the page, that is—measurements are made in a

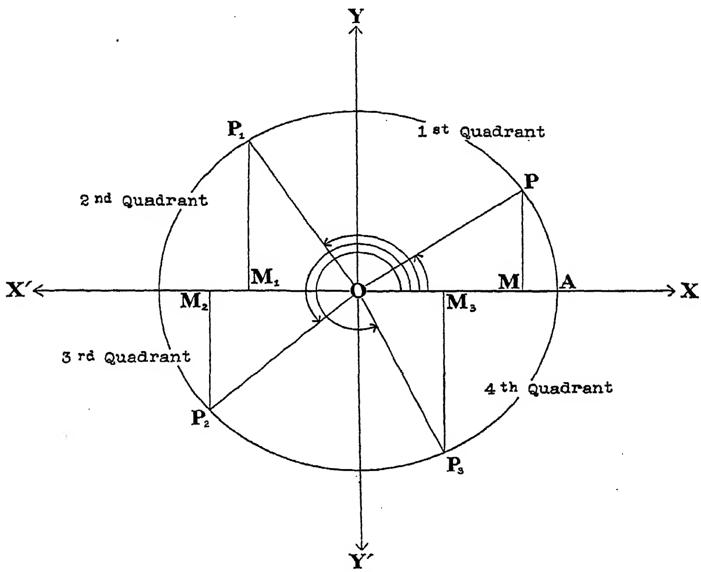


FIGURE 156.

positive direction and carry a plus sign, and that along OX' or OY'—to the left or towards the bottom of the page—they are made in a *negative* direction and carry a minus sign.

The radius OP is considered to be always positive and to move anti-clockwise round the circle, occupying successive positions P, P_1, P_2, and P_3 in the four quadrants into which the axes divide the circle, and forming the angles AOP, AOP_1, AOP_2 and AOP_3.

(1) *First Quadrant.* In this quadrant OM and MP are measured in positive directions, and since OP is positive, all the trigonometrical functions must be positive.

(2) *Second Quadrant.* In this quadrant M_1P_1 is positive, but OM_1 is negative. Sin AOP_1 is therefore positive, and cos AOP_1 is

negative. Hence the tangent, cotangent and secant of angles on this quadrant are negative, and the cosecant is positive.

(3) *Third Quadrant.* In this quadrant both M_2P_2 and OM_2 are negative, and both $\sin AOP_2$ and $\cos AOP_2$ are negative. The tangent is therefore positive.

(4) *Fourth Quadrant.* In this quadrant, M_3P_3 is negative but OM_3 is positive. Hence the sine is negative; the cosine is positive; and the tangent is negative.

In summary these facts may be stated thus:

First Quadrant—all functions positive.
Second Quadrant—only the sine and cosecant positive.
Third Quadrant—only the tangent and cotangent positive.
Fourth Quadrant—only the cosine and secant positive.

Complementary Angles. Angles that, when added together, make 90°, are said to be *complementary*. Thus if one angle is 34°, the complementary angle is 56°.

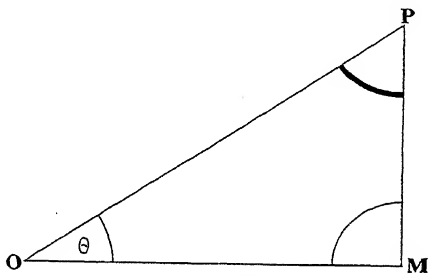


FIGURE 157.

In any right-angled triangle the two acute angles are complementary since the sum of the three angles, of which one is 90°, must be 180°. Figure 157 shows this fact. and also that:

$$\text{in }\theta=\frac{MP}{OP}=\cos(90°-\theta)$$

$$\cos\theta=\frac{OM}{OP}=\sin(90°-\theta)$$

Supplementary Angles. Angles that, when added together, make 180°, are said to be *supplementary*. Thus if one angle is 34°, the supplementary angle is 146°.

In figure 158, MOP and MOP' are supplementary angles. Then, since the angle $M'OP'$ is numerically equal to the angle MOP:

$$\sin(180°-\theta)=\frac{M'P'}{OP'}=\frac{MP}{OP}=\sin\theta$$

$$\cos(180°-\theta)=\frac{OM'}{OP'}=\frac{-OM}{OP}=-\cos\theta$$

$$\therefore\ \tan(180°-\theta)=-\tan\theta$$

By reference to a geometrical figure, or by remembering the signs that the various functions take in the separate quadrants, it can be seen that :

$$\sin(90°+\theta)=\cos\theta \qquad \cos(90°+\theta)=-\sin\theta$$
$$\sin(180°+\theta)=-\sin\theta \qquad \cos(180°+\theta)=-\cos\theta$$
$$\sin(360°-\theta)=-\sin\theta \qquad \cos(360°-\theta)=\cos\theta$$

Since $(360°-\theta)$ is, in effect, the negative angle, $-\theta$, the last formulæ may be written :

$$\sin(-\theta)=-\sin\theta \qquad \cos(-\theta)=\cos\theta$$

Connexion between the Sine and Cosine. It has been shown that the tangent, cotangent, secant and cosecant can be expressed in terms of the sine and cosine. The sine and cosine are themselves connected by a relation which can be established geometrically.

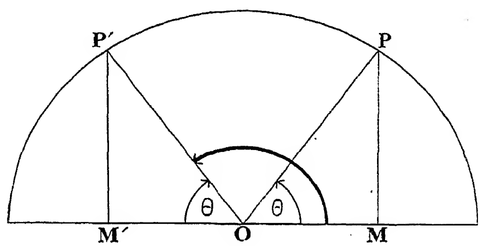


FIGURE 158.

By the Theorem of Pythagoras, the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. Therefore, in figure 157 :

$$MP^2+OM^2=OP^2$$

i.e.
$$\frac{MP^2}{OP^2}+\frac{OM^2}{OP^2}=1$$

or
$$\sin^2\theta+\cos^2\theta=1$$

This relation may be written :

$$\tan^2\theta+1=\sec^2\theta$$
$$1+\cot^2\theta=\operatorname{cosec}^2\theta$$

These formulæ hold for all values of θ, because the square of the trigonometrical function is always positive although the angle may be such that the function itself is negative.

The Plane Triangle. There are several formulæ connecting the sides and angles of a triangle, and the choice of formula is governed, as a rule, by convenience.

The convention adopted in order to distinguish the sides and

angles of a triangle ABC—figure 159—is that 'A' shall denote the angle at A where AB cuts AC, and that 'a' shall denote the side BC opposite the angle A. A like significance attaches to B, b, C and c.

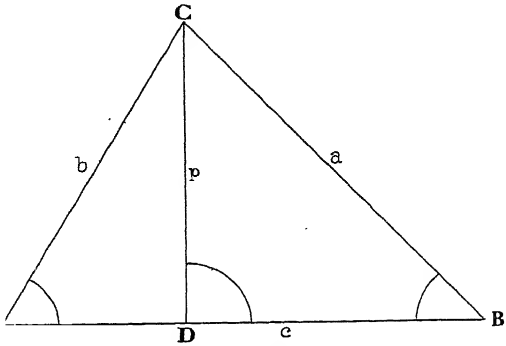


FIGURE 159.

The Sine Formula. This formula is established by dropping a perpendicular from any vertex on to the opposite side. In figure 159, the perpendicular is CD, denoted by p. Then :

$$\sin A = p/b \qquad \sin B = p/a$$

i.e.
$$p = b \sin A \qquad p = a \sin b$$

$\therefore$
$$b \sin A = a \sin B$$

or
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

If a perpendicular is dropped from A to BC, it can be shown that :

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Hence :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

If two angles and one side of the triangle are given, the third angle is known to be $[180° - (A + B)]$, and the sine formula gives the remaining sides.

The formula is true for an obtuse-angled triangle.

Figure 160 shows that ambiguity arises if the formula is used for solving the triangle when two sides and an angle other than the included angle are given, the given angle being opposite the smaller side. If, for example, the sides b and c and the angle C are given, the angle found from the formula is either ABC or its supplement $AB'C$, because the sine of an angle is equal to the sine of its supplement.

The Cosine Formula. This formula is established by applying the Theorem of Pythagoras to the right-angled triangles ADC and BDC in figure 159. Thus:

$$b^2=p^2+AD^2 \qquad\qquad a^2=p^2+BD^2$$

$$\begin{aligned}\therefore \quad a^2&=(b^2-AD^2)+BD^2\\ &=b^2-AD^2+(c-AD)^2\\ &=b^2-AD^2+c^2-2cAD+AD^2\\ &=b^2+c^2-2cAD\\ &=b^2+c^2-2bc\cos A\end{aligned}$$

In the same way it can be established that:

$$b^2=c^2+a^2-2ca\cos B$$
$$c^2=a^2+b^2-2ab\cos C$$

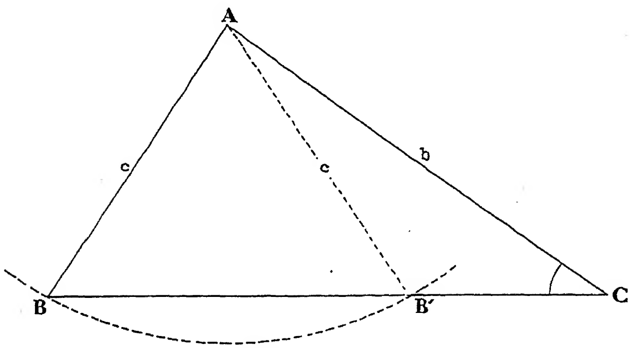


FIGURE 160.

This formula is true for any triangle, but it must be remembered that, if the angle A, B or C is greater than 90°, the angle lies in the second quadrant and its cosine is negative.

The formula gives the third side when two sides and the included angle are known.

The Tangent Formula. The sine formula can be adjusted algebraically and written:

$$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$$

There are similar formulæ in b and c, and in c and a.

The Area of a Triangle. It is known that the area of a triangle is equal to half the base multiplied by the perpendicular height. The area of the triangle ABC—figure 159—is therefore given by:

$\frac{1}{2}ab\sin C$ . . . . . . (1)
$\frac{1}{2}bc\sin A$ . . . . . . (2)
$\frac{1}{2}ca\sin B$ . . . . . . (3)

Functions of the Sum and Difference of Two Angles. It can be shown that the following trigonometrical relations connect the sum and difference of two angles A and B:

$$\sin(A+B)=\sin A\cos B+\cos A\sin B \quad . \quad . \quad (1)$$
$$\cos(A+B)=\cos A\cos B-\sin A\sin B \quad . \quad . \quad (2)$$
$$\sin(A-B)=\sin A\cos B-\cos A\sin B \quad . \quad . \quad (3)$$
$$\cos(A-B)=\cos A\cos B+\sin A\sin B \quad . \quad . \quad (4)$$

Functions of the Half Angle. If A is equal to B, then it follows from formulæ (1) and (2) that:

$$\begin{aligned}\sin 2A &= 2\sin A\cos A\\ \cos 2A &= \cos^2 A-\sin^2 A\\ &= 1-2\sin^2 A\\ &= 2\cos^2 A-1\end{aligned}$$

In terms of the half angle these formulæ are:

$$\begin{aligned}\sin A &= 2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A\\ \cos A &= \cos^2\tfrac{1}{2}A-\sin^2\tfrac{1}{2}A\\ &= 1-2\sin^2\tfrac{1}{2}A\\ &= 2\cos^2\tfrac{1}{2}A-1\end{aligned}$$

Sum and Difference of Functions. Formulæ (1), (2), (3) and (4), which relate to *the sines and cosines of sums and differences*, may be combined to give other formulæ which relate to *the sums and differences of sines and cosines.*

By adding (1) and (3), and writing P for $(A+B)$ and Q for $(A-B)$ so that A is equal to $\frac{1}{2}(P+Q)$ and B to $\frac{1}{2}(P-Q)$:

$$\sin(A+B)+\sin(A-B)=2\sin A\cos B$$

i.e. $$\sin P+\sin Q=2\sin\tfrac{1}{2}(P+Q)\cos\tfrac{1}{2}(P-Q)$$

By subtracting (3) from (1):

$$\sin(A+B)-\sin(A-B)=2\cos A\sin B$$

i.e. $$\sin P-\sin Q=2\cos\tfrac{1}{2}(P+Q)\sin\tfrac{1}{2}(P-Q)$$

By using formulæ (2) and (4), it can be shown that:

$$\cos P+\cos Q=2\cos\tfrac{1}{2}(P+Q)\cos\tfrac{1}{2}(P-Q)$$
$$\cos P-\cos Q=-2\sin\tfrac{1}{2}(P+Q)\sin\tfrac{1}{2}(P-Q)$$

The Sine and Cosine Curves. Figure 156, on page 239, shows that as an angle increases from 0° to 90°, the perpendicular opposite to it, MP, increases from 0 to the length of the radius of circle described, and that the projection of this radius, OM, decreases from the given length to 0. Sin θ therefore increases from 0 to 1, and cos θ decreases from 1 to 0.

Between 90° and 180°, sin θ decreases from 1 to 0, and cos θ from 0 to -1.

Between 180° and 270°, sin θ decreases from 0 to -1, and cos θ increases from -1 to 0.

Between 270° and 360°, sin θ increases from -1 to 0, and cos θ from 0 to 1.

After 360°, the cycle is repeated.

Figure 161 shows the curves $y=\sin\theta$ and $y=\cos\theta$, plotted for values of θ between 0° and 360°.

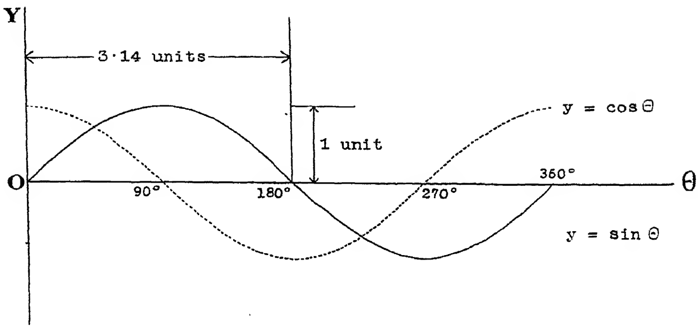


FIGURE 161.

The Haversine. As explained in Chapter IV, the quantity $\frac{1}{2}(1-\cos\theta)$ is known as the haversine of θ. *Inman's Tables* give the values of the function for values of θ from 0° to 180°. For values of θ greater than 180°:

$$\begin{aligned}\text{hav }\theta &= \tfrac{1}{2}(1-\cos\theta)\\ &= \tfrac{1}{2}[1-\cos(360°-\theta)]\\ &= \text{hav }(360°-\theta)\end{aligned}$$

i.e. $$\text{hav }210° = \text{hav }150°$$

Since hav θ is equal to $\frac{1}{2}(1-\cos\theta)$, and $\cos\theta$ cannot be greater than 1, hav θ must always be positive.

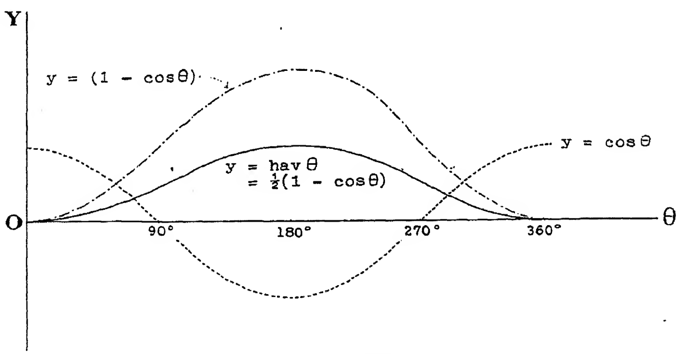


FIGURE 162.

Figure 162 shows the haversine curve in relation to the cosine curve from which it is derived.

The haversine is used in navigation because :

(1) between 0° and 180° it is a continuously increasing and therefore a 'single-valued' function, and there is thus no ambiguity in deciding whether an angle is greater or less than 90°.

(2) it is always positive and therefore suitable for logarithmic calculation.

The above advantages also apply to the versine, but the haversine is preferable because its limiting values, 0 and 1, make the construction of logarithmic and other tables comparatively simple, whereas the limiting values of the versine, 0 and 2, do not.

The Sine of a Small Angle. Certain approximations suggest themselves when the angle is small.

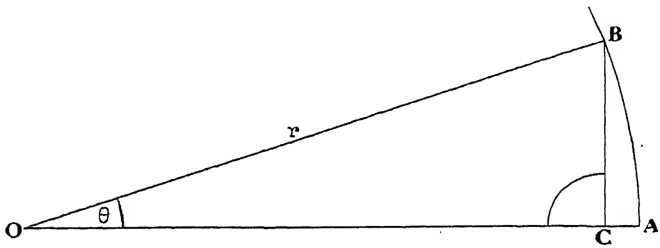


FIGURE 163.

In figure 163, AOB is a small angle θ, measured in radians. AB is the arc of a circle which subtends this small angle. The radius of the circle is r, and BC is perpendicular to OA at C.

On page 238 it was stated that the length of arc of a circle is equal to the radius multiplied by the angle subtended in radians. That is :

$$AB = r \times \theta$$

or

$$\theta = \frac{AB}{r}$$

But

$$\sin\theta = \frac{BC}{r}$$

Therefore, when θ is sufficiently small for AB to approximate to BC :

$$\sin\theta = \theta$$

If there are x' in this small angle of θ radians, then there must be $(x/\theta)'$ in one radian. But one radian is equal to 57°17′45″, or approximately 3,438′. Hence :

$$\frac{x}{\theta} = 3,438$$

i.e.

$$\overline{3,438}$$

The relation, $\sin \theta = \theta$, therefore becomes :

$$\sin x' = \frac{x}{3{,}438}$$

Since this relation holds for any value of x that is small :

$$\sin 1' = \frac{}{3}$$

$$\therefore \qquad \sin x' = x \sin 1'$$

These adjustments are important when practical results have to be obtained from theoretical calculation, as in the construction of the ex-meridian tables described in Volume III.

The Cosine of a Small Angle. Figure 163 shows that when θ is small, OC approximates to OA, which is the same as OB. But :

$$\cos \theta = \frac{OC}{OB}$$

Therefore, when θ is small, $\cos \theta$ is equal to 1.

A second approximation can be obtained if $\cos \theta$ is expressed in terms of the half angle, for then :

$$\cos \theta = 1 - 2 \sin^2 \tfrac{1}{2}\theta$$

$$\text{i.e.} \qquad \cos \theta = 1 - 2 \left(\tfrac{1}{2}\theta\right)^2$$

$$\therefore \qquad \cos \theta = 1 - \tfrac{1}{2}\theta^2$$

PART II

A SUMMARY OF SPHERICAL TRIGONOMETRY

The construction of the spherical triangle and its solution by the fundamental formula :

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

—was described in Chapter IV.

The formula suffices for the cosine-haversine method of finding the intercept when a position line is drawn according to the Marc St. Hilaire principle. Other formulæ, however, are necessary for other methods.

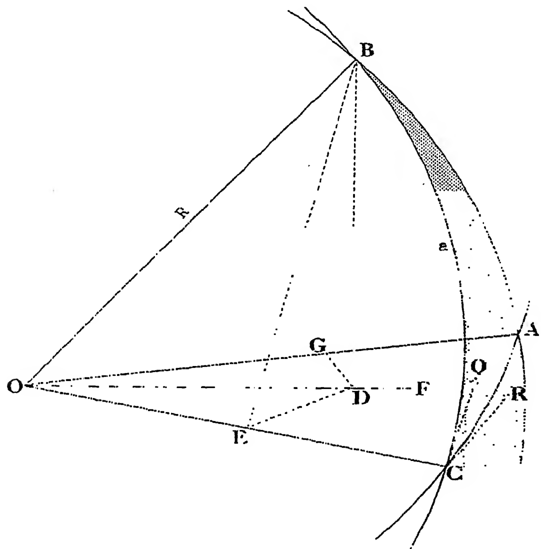


FIGURE 164.

The Sine Formula. This is analogous to the sine formula in plane trigonometry, and it can be usefully modified for the solution of triangles in which one side or one angle is equal to 90°. The formula is :

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

In figure 164, O is the centre of the sphere on the surface of which AB, BC and CA are the great-circle arcs forming the spherical triangle ABC. BD is perpendicular to the plane OAC. DE is perpendicular to OC. The triangles BOD and BED are thus right-angled at D. Hence :

$$OB^2=OD^2+BD^2 \quad \ldots\ldots \quad (1)$$
$$BE^2=BD^2+ED^2 \quad \ldots\ldots \quad (2)$$
$$OD^2=OE^2+ED^2 \quad \ldots\ldots \quad (3)$$

From (1) and (2), by subtraction :

$$OB^2-BE^2=OD^2-ED^2$$

From (3), by substitution :

$$OB^2-BE^2=OE^2+ED^2-ED^2=OE^2$$

i.e. $$OB^2=BE^2+OE^2$$

This relation shows that the triangle BOE is right-angled at E. BE is therefore perpendicular to OC.

In the spherical triangle ABC, CQ is the tangent to the great-circle arc CB at C, and CR is the tangent to the great-circle arc CA at C. The angle QCR therefore measures the spherical angle BCA.

CQ, being perpendicular to OC and lying in the plane OBC, is parallel to EB because EB is also perpendicular to OC and lies in the plane OBC. For similar reasons CR is parallel to ED. Hence :

$$\angle BED=\angle QCR=C$$

Also, since the triangle BED is right-angled at D :

$$BD=BE \sin C$$

But from the right-angled triangle BOE :

$$BE=OB \sin a=R \sin a$$

$\therefore$ $$BD=R \sin a \sin C$$

By drawing DG perpendicular to OA, it can be proved that :

$$BD=R \sin c \sin A$$

$\therefore$ $$R \sin a \sin C=R \sin c \sin A$$

i.e. $$\frac{\sin a}{\sin A}=\frac{\sin c}{\sin C}$$

Clearly, if a perpendicular were drawn from A on to the plane OBC, it could be proved that :

$$\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$$

Hence the complete formula :

$$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$$

In the triangle drawn in figure 164, all angles and sides are less than 90°. It can be shown, however, by modifying the proof slightly, that the formula is true for all triangles.

The sine formula in spherical trigonometry has the same limitation as its counterpart in plane trigonometry in that ambiguity arises if it is used to solve the triangle when two sides and one angle are given.

Suppose, for example, that a is 66°, b is 50° and B is 40°. From the formula :

$$\begin{aligned} \sin A &= \frac{\sin a}{\sin b} \times \sin B \\ &= \sin a \sin B \operatorname{cosec} b \\ &= \sin 66° \sin 40° \operatorname{cosec} 50° \\ &= \cdot 76657 \\ \therefore \quad A &= 50°02'\cdot 8 \text{ or } 129°57'\cdot 2 \end{aligned}$$

Figure 165 shows that $A'BC$ and $A''BC$ are possible triangles, and until further information is given this ambiguity must, as a

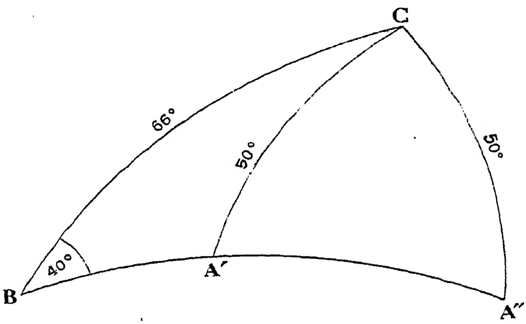


FIGURE 165.

rule, remain. Sometimes, however, it can be removed by remembering that the larger side faces the larger angle—a fact which does not assist in the example given because both 50°02′·8 and 129°57′·2, the angles opposite the side 66°, are greater than 40°, the angle opposite the side 50°.

Right-Angled Triangles. If one angle of a spherical triangle is a right angle, the formulæ for solving the triangle are greatly simplified.

Thus, if the angle C in the triangle ABC is a right angle, the fundamental formula becomes :

$$\cos c = \cos a \cos b$$

and the sine formula becomes :

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \sin c$$

The numerous formulæ thus obtainable are best summarised by Napier's Rules.

Napier's Mnemonic Rules for Right-Angled Triangles. In these rules the right-angled spherical triangle is considered in terms of five ' circular parts '.

If C is the right angle, these parts are a, b, $(90°-A)$, $(90°-B)$ and $(90°-c)$ and for convenience they are shown as sectors of a circle. (Figure 166.)

The circular parts are arranged in cyclic order, starting from a vertical radius which represents the right angle. Sides and angles thus alternate, and the sector representing any side appears in the

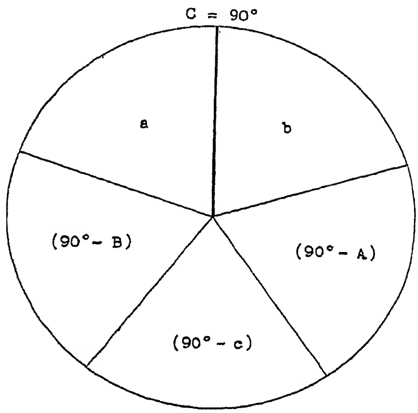


FIGURE 166.

circle opposite the sector representing the angle, just as the side itself appears in the triangle.

The rules themselves are stated in terms of *middle, adjacent* and *opposite* parts :

sin middle=product of tangents of adjacents
sin middle=product of cosines of opposites

Any part can be taken as a ' middle '. The ' adjacents ', as the name suggests, lie one on each side. The ' opposites ' are the two other parts. Thus, by the first rule :

$$\sin(90°-B)=\tan a \tan(90°-c)$$

i.e. $$\cos B=\tan a \cot c$$

By the second rule :

$$\sin(90°-B)=\cos b \cos(90°-A)$$

i.e. $$\cos B=\cos b \sin A$$

Quadrantal Triangles. If one *side* of the spherical triangle is 90°, the triangle is said to be *quadrantal*, and the formulæ for its solution are again simplified.

If the side c is 90°, the fundamental formula gives :

$$\cos 90° = \cos a \cos b + \sin a \sin b \cos C$$

i.e. $$\cos a \cos b + \sin a \sin b \cos C = 0$$

or $$\cos C = -\cot a \cot b$$

It also gives :

$$\cos a = \cos b \cos 90° + \sin b \sin 90° \cos A$$

i.e. $$\cos a = \sin b \cos A$$

and $$\cos b = \cos 90° \cos a + \sin 90° \sin a \cos B$$

i.e. $$\cos b = \sin a \cos B$$

The sine formula gives :

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \sin C$$

These results may be combined to give other formulæ.

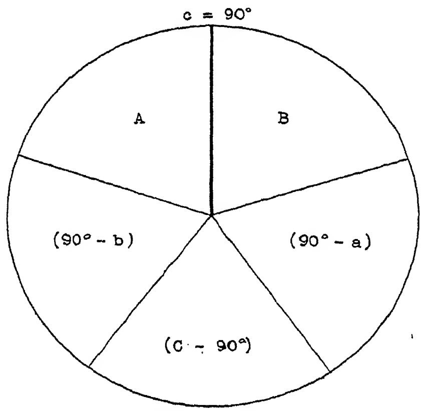


FIGURE 167.

Napier's Mnemonic Rules for Quadrantal Triangles. A simple adjustment makes the rules for the right-angled triangle apply to the quadrantal. The circular parts for the triangle ABC, in which the side c is 90°, are A, B, $(90°-a)$, $(90°-b)$ and $(C°-90°)$, and they are arranged as shown in figure 167.

The rules themselves remain unaltered.

The ' Four Part ' Formula. This is a formula the terms of which are four consecutive angles and sides of any spherical triangle.

In figure 168, the four parts to be considered are C, a, B and c. The angle B, contained by the two sides a and c, is called the ' inner angle ' or the ' I.A.'. The side a, common to the angles B and C.

is called the 'inner side' or the 'I.S.'. The others are the 'other angle' C, denoted by 'O.A.', and the 'other side' c, denoted by 'O.S.'.

The formula states that :

$$\underline{\cos(\text{I.S.})\cos(\text{I.A.}) = \sin(\text{I.S.})\cot(\text{O.S.}) - \sin(\text{I.A.})\cot(\text{O.A.})}$$

It may be proved thus :

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

By substituting for $\cos b$:

$$\cos c = \cos a\,(\cos c \cos a + \sin c \sin a \cos B) + \sin a \sin b \cos C$$

i.e. $\cos c = \cos c\,(1 - \sin^2 a) + \sin a \cos a \sin c \cos B + \sin a \sin b \cos C$

Therefore, since $\cos c$ cancels and $\sin a$ is common to the remaining terms :

$$\sin a \cos c = \cos a \sin c \cos B + \sin b \cos C$$

i.e. $\sin a \dfrac{\cos c}{\sin c} = \cos a \cos B + \dfrac{\sin b}{\sin c} \cos C$

Hence, by the sine formula :

$$\sin a \cot c = \cos a \cos B + \frac{\sin B}{\sin C} \cos C$$

i.e. $\cos a \cos B = \sin a \cot c - \sin B \cot C$

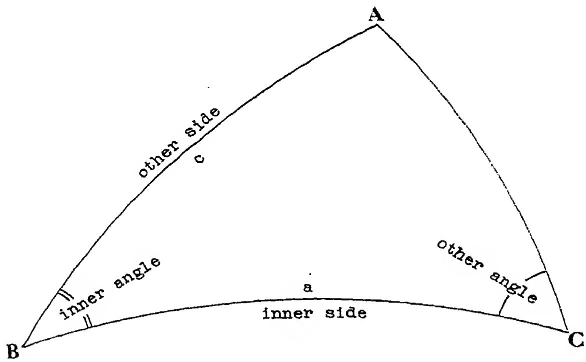


FIGURE 168.

Delambre's and Napier's Analogies. There are a number of formulæ for solving the spherical triangle which are analogous to those for solving the plane triangle.

Delambre's analogies are :

$$\sin \tfrac{1}{2}c \sin \tfrac{1}{2}(A - B) = \cos \tfrac{1}{2}C \sin \tfrac{1}{2}(a - b) \quad . \quad . \quad (1)$$
$$\sin \tfrac{1}{2}c \cos \tfrac{1}{2}(A - B) = \sin \tfrac{1}{2}C \sin \tfrac{1}{2}(a + b) \quad . \quad . \quad (2)$$
$$\cos \tfrac{1}{2}c \sin \tfrac{1}{2}(A + B) = \cos \tfrac{1}{2}C \cos \tfrac{1}{2}(a - b) \quad . \quad . \quad (3)$$
$$\cos \tfrac{1}{2}c \cos \tfrac{1}{2}(A + B) = \sin \tfrac{1}{2}C \cos \tfrac{1}{2}(a + b) \quad . \quad . \quad (4)$$

Napier's analogies can be obtained by dividing these formulæ in appropriate pairs. Thus :

$$\tan \tfrac{1}{2}(a+b)=\frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \tfrac{1}{2}c \quad . \quad . \quad \text{from (2) and (4)}$$

$$\tan \tfrac{1}{2}(a-b)=\frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \tfrac{1}{2}c \quad . \quad . \quad \text{from (1) and (3)}$$

$$\tan \tfrac{1}{2}(A+B)=\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2}C \quad . \quad . \quad \text{from (3) and (4)}$$

$$\tan \tfrac{1}{2}(A-B)=\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2}C \quad . \quad . \quad \text{from (1) and (2)}$$

PART III

COMPARISON OF METHODS OF WORKING SIGHTS

The Sun sight worked by different methods in Chapters XIV, XV and XVI of this volume is here reproduced and the working of the three methods shown in parallel columns.

At Z.T. 0750(+2), *3rd April* 1937, *the D.R. was* 34°31′*N.*, 28°36′*W.*, *and the deck watch showed* $9^h51^m33^s$ *when the sextant altitude of the Sun's lower limb was* 26°18′·0. *The deck watch was* 31^s *slow on G.M.T.; the index error was* −2′·6 ; *and the height of eye* 32 *feet. Draw the position line.*

The following work is common to the three methods.

		Sun's Declination
Z.T.	0750 3rd April	5°11′·7N.
Zone	+2	+1′·8
G.D.	0950 3rd April	5°13′·5N.

	h m s	E h m s
D.W.	9 51 33	11 56 32·2
Error slow	31	+1·4
G.M.T.	9 52 04 3rd April	11 56 33·6

Sext. Alt.	26°18′·0
I.E.	−2′·6
Obs. Alt.	26°15′·4
Corr^n.	+8′·6
True Alt.	26°24′·0

COSINE-HAVERSINE METHOD

(Chapter XIV)

	h m s	
G.M.T.	9 52 04 3rd April	
Long. W.	1 54 24	
L.M.T.	7 57 40	
E	11 56 34	
H.A.T.S.	19 54 14	
Lat. N.	34°31′·0	
Dec. N.	5°13′·5	
	29°17′·5	
		9·416 60
		9·915 91
		9·998 20
		9·330 71
		·214 15
		·063 93
		·278 08
C.Z.D.	63°39′·1	
T.Z.D.	63°36′·0	
Intercept	3′·1 towards	

Azimuth

PZ	55°29′·0	0·084 09
ZX	63°39′·1	0·047 63
PX	84°46′·5	
	92°56′·6	4·860 36
	76°36′·4	4·792 27
LPZX =	102°32′·9	9·784 35

Intercept plotted from :
{ 34°31′·0N.
28°36′·0W.

S.A.N. TABLES

(Chapter XV)

	h m s
G.M.T.	9 52 04 3rd April
E	11 56 34
G.H.A.	21 48 38
=	327°09′·5
Long. W.	28°09′·5
H.A.T.S.	299°W. or 61°E.
Assumed lat.	35°N.
Dec.	5°13′·5N.
K	55°18′·1N.
K∼*d*	50°04′·6
A	15637
B	19263
A+B	34900
Calc. Alt.	26°35′·8
True Alt.	26°24′·0
Intercept	11′·8 away
D	145
E	78
D+E	223
Z_1	+44°·0
Z_2	+59°·1
	N.103°·1E.

35°00′·0N.
28°09′·5W.

ALTITUDE-AZIMUTH TABLES

(Chapter XVI)

	h m s		h m s
G.M.T.	9 52 04 3rd April	G.M.T.	9 52 04 3rd April
E	11 56 34	E	11 56 34
G.H.A.	21 48 38	G.H.A.	21 48 38
	327°09′·5		327°09′·5
Long. W.	28°09′·5	Long. W.	28°36′·0
H.A.T.S.	299°W. or 61°E.	H.A.T.S.	298°33′·5
			= 61°26′·5E.
Assumed lat.	35°N.	Assumed lat.	35°N.
△d	60	△d	60
difference	13′·5	difference	13′·5
		△h	80
		difference	26′·6
Altitude	26°27′·7	Altitude	26°27′·7
		Corrⁿ. △d	+8′·1
		Corrⁿ. △h	−21′·2
Calc. Alt.	26°35′·8	Calc. Alt.	26°14′·6
True Alt.	26°24′·0	True Alt.	26°24′·0
Intercept	11′·8 away	Intercept	9′·4 towards
	N.103°E.		N.103°E.
	35°00′·0N. 28°09′·5W.		35°00′·0N. 28°36′·0W.

PART IV

EXTRACTS FROM THE ABRIDGED *NAUTICAL ALMANAC*

APRIL 1937

Date	Day of Year	Sun's Semi-diameter	THE MOON'S Semi-diameter	THE MOON'S Horizontal Parallax	THE MOON'S Meridian Passage Upper		THE MOON'S Meridian Passage Lower		THE MOON'S Age at 0h
		′	′	′	h m	m	h m	m	d
Thur. 1	91	16·0	14·8	54·2	03 48	49	12 12	50	19·2
Fri. 2	92	16·0	14·8	54·4	04 37	50	17 02	50	20·2
Sat. 3	93	16·0	14·9	54·7	05 27	49	17 52	49	21·2
Sun. 4	94	16·0	15·1	55·3	06 16	49	18 41	49	22·2
Mon. 5	95	16·0	15·2	55·9	07 05	49	19 30	49	23·2

THE SUN

G.M.T.	R	Dec.	
h	h m s	° ′	h m s
Friday 2			
00	12 39 40·8	N. 4 40·9	11 56 08·4
02	12 40 00·5	4 42·8	11 56 09·9
04	12 40 20·2	4 44·8	11 56 11·4
06	12 40 40·0	4 46·7	11 56 12·9
08	12 40 59·7	4 48·6	11 56 14·4
10	12 41 19·4	4 50·6	11 56 15·9
12	12 41 39·1	4 52·5	11 56 17·4
14	12 41 58·8	4 54·4	11 56 18·9
16	12 42 18·5	4 56·3	11 56 20·4
18	12 42 38·3	4 58·3	11 56 21·9
20	12 42 58·0	5 00·2	11 56 23·3
22	12 43 17·7	5 02·1	11 56 24·8
Saturday 3			
00	12 43 37·4	N. 5 04·0	11 56 26·3
02	12 43 57·1	5 05·9	11 56 27·8
04	12 44 16·8	5 07·8	11 56 29·3
06	12 44 36·5	5 09·8	11 56 30·8
08	12 44 56·2	5 11·7	11 56 32·2
10	12 45 15·9	5 13·6	11 56 33·7
12	12 45 35·7	5 15·5	11 56 35·2
14	12 45 55·4	5 17·4	11 56 36·7
16	12 46 15·1	5 19·3	11 56 38·2
18	12 46 34·8	5 21·3	11 56 39·7
20	12 46 54·5	5 23·2	11 56 41·1
22	12 47 14·2	5 25·1	11 56 42·6
Sunday 4			
00	12 47 33·9	N. 5 27·0	11 56 44·1
02	12 47 53·6	5 28·9	11 56 45·6
04	12 48 13·3	5 30·8	11 56 47·1
06	12 48 33·1	5 32·8	11 56 48·6
08	12 48 52·8	5 34·7	11 56 50·0
10	12 49 12·5	5 36·6	11 56 51·5
12	12 49 32·2	5 38·5	11 56 53·0
14	12 49 51·9	5 40·4	11 56 54·5
16	12 50 11·6	5 42·3	11 56 55·9
18	12 50 31·4	5 44·2	11 56 57·4
20	12 50 51·1	5 46·1	11 56 58·9
22	12 51 10·8	5 48·0	11 57 00·3
24	12 51 30·5	N. 5 49·9	11 57 01·8

MOON'S RIGHT ASCENSION AND DECLINATION

G.M.T.	R.A.		Dec.	
h	h m s		° ′	
Friday 2				
00	17 07 35	258	22 50·2	0·1
02	17 11 53	259	22 50·1	0·5
04	17 16 12	259	22 49·6	1·0
06	17 20 31	259	22 48·6	1·5
08	17 24 50	259	22 47·1	1·9
10	17 29 09	259	22 45·2	2·3
12	17 33 28	259	22 42·9	2·7
14	17 37 47	260	22 40·2	3·2
16	17 42 07	259	22 37·0	3·6
18	17 46 26	259	22 33·4	4·1
20	17 50 45	260	22 29·3	4·5
22	17 55 05	260	22 24·8	5·0
Saturday 3				
00	17 59 25	259	22 19·8	5·3
02	18 03 44	260	22 14·5	5·9
04	18 08 04	259	22 08·6	6·2
06	18 12 23	260	22 02·4	6·7
08	18 16 43	259	21 55·7	7·1
10	18 21 02	260	21 48·6	7·6
12	18 25 22	259	21 41·0	8·0
14	18 29 41	260	21 33·0	8·4
16	18 34 01	259	21 24·6	8·9
18	18 38 20	259	21 15·7	9·2
20	18 42 39	259	21 06·5	9·7
22	18 46 58	259	S. 20 56·8	10·2
Sunday 4				
00	18 51 17	259	S. 20 46·6	10·5
02	18 55 36	259	20 36·1	11·0
04	18 59 55	258	20 25·1	11·4
06	19 04 13	259	20 13·7	11·8
08	19 08 32	258	20 01·9	12·2
10	19 12 50	258	19 49·7	12·7
12	19 17 08	258	19 37·0	13·0
14	19 21 26	258	19 24·0	13·5
16	19 25 44	258	19 10·5	13·8
18	19 30 02	257	18 56·7	14·3
20	19 34 19	257	18 42·4	14·7
22	19 38 36	258	18 27·7	15·0
24	19 42 54		18 12·7	

Date	VENUS R.A.		VENUS Dec.		VENUS Meridian Passage	MARS R.A.		MARS Dec.		MARS Meridian Passage
	h m s	s	° ′	′	h m	h m s	s	° ′	′	h m
Thur. 1	02 02 20	50	N. 20 13·8	1·5	13 24	16 10 42	36	S. 20 04·1	2·9	03 35
Fri. 2	02 01 30	59	20 12·3	2·8	13 19	16 11 18	34	20 07·0	2·8	03 31
Sat. 3	02 00 31	67	20 09·5	4·2	13 14	16 11 52	31	20 09·8	2·7	03 28

Date	JUPITER R.A.		JUPITER Dec.		JUPITER Meridian Passage	SATURN R.A.		SATURN Dec.		SATURN Meridian Passage
	h m s	s	° ′	′	h m	h m s	s	° ′	′	h m
Thur. 1	19 45 06	32	S. 21 20·2	1·2	07 08	23 52 50	27	S. 2 57·9	2·8	11 16
Fri. 2	19 45 38	31	21 19·0	1·2	07 05	23 53 17	26	2 55·1	2·8	11 12
Sat. 3	19 46 09	31	21 17·8	1·2	07 01	23 53 43	27	2 52·3	2·8	11 09

PART V

EXTRACTS FROM THE *AIR ALMANAC*

OCTOBER 1937

STARS, 1937 OCTOBER 1 and 2

No.	Name	S.H.A.	Dec.	G.M.T.	G.H.A. of First Point of Aries=*A* 00m	10m	20m	30m	40m	50m
					Friday, October 1					
		° ′	° ′	h	° ′	° ′	° ′	° ′	° ′	° ′
1	ACHERNAR	336 08	S. 57 33	00	9 18	11 49	14 19	16 50	19 20	21 51
2	ACRUX	174 14	S. 62 45	01	24 21	29 51	29 22	31 52	34 23	36 53
3	*Adara*	255 57	S. 28 53	02	39 23	41 54	44 24	46 55	49 25	51 55
4	ALDEBARAN	291 54	N. 16 23	03	54 26	56 56	59 27	61 57	64 28	56 58
5	*Alhena*	261 28	N. 16 27	04	69 28	71 29	74 29	77 00	79 30	82 00
6	*Alioth*	167 11	N. 56 18	05	84 31	87 01	89 32	92 02	94 32	97 03
7	*Alnitak*	275 35	S. 1 58	06	99 33	102 04	104 34	107 04	109 35	112 05
8	*Alphacca*	126 59	N. 26 56	07	114 36	117 06	119 37	122 07	124 37	127 08
9	*Alphard*	218 52	S. 8 23	08	129 38	132 09	134 39	137 09	139 40	142 10
10	*Alpheratz*	358 42	N. 28 45	09	144 41	147 11	149 41	152 12	154 42	157 13
11	ALTAIR	63 03	N. 8 42	10	159 43	162 14	164 44	167 14	169 45	172 15
12	*Anilam*	276 44	S. 1 14	11	174 46	177 16	179 46	182 17	184 47	187 18
13	ANTARES	113 36	S. 26 18	12	189 48	192 18	194 49	197 19	199 50	202 20
14	ARCTURUS	146 48	N. 19 30	13	204 50	207 21	209 51	212 22	214 52	217 23
15	γ Argus	238 06	S. 47 09	14	219 53	222 23	224 54	227 24	229 55	232 25
16	ε Argus	234 42	S. 59 18	15	234 55	237 26	239 56	242 27	244 57	247 27
17	*Bellatrix*	279 33	N. 6 18	16	249 58	252 28	254 59	257 29	260 00	262 30
18	*Benetnasch*	153 44	N. 49 37	17	265 00	267 31	270 01	272 32	275 02	277 32
19	BETELGEUSE	272 03	N. 7 24	18	280 03	282 33	285 04	287 34	290 04	292 35
20	δ Canis Maj.	253 32	S. 26 17	19	295 05	297 36	300 06	302 37	305 07	307 37
21	CANOPUS	264 21	S. 52 39	20	310 08	312 38	315 09	317 39	320 09	322 40
22	CAPELLA	281 58	N. 45 56	21	325 10	327 41	330 11	332 41	335 12	337 42
23	*Castor*	247 20	N. 32 01	22	340 13	342 43	345 13	347 44	350 14	352 45
24	β CENTAURI	150 09	S. 60 05	23	355 15	357 46	0 16	2 46	5 17	7 47
25	β CRUCIS	168 59	S. 59 21		**Saturday, October 2**					
26	γ Crucis	173 05	S. 56 46	00	10 18	12 48	15 18	17 49	20 19	22 50
27	DENEB	50 10	N. 45 04	01	25 20	27 50	30 21	32 51	35 22	37 52
28	*Denebola*	183 32	N. 14 55	02	40 23	42 53	45 23	47 54	50 24	52 55
29	*Diphda*	349 52	S. 18 19	03	55 25	57 55	60 26	62 56	65 27	67 57
30	*Dubhe*	195 02	N. 62 05	04	70 27	72 58	75 28	77 59	80 29	83 00
31	FOMALHAUT	16 26	S. 29 57	05	85 30	88 00	90 31	93 01	95 32	98 02
32	*Hamal*	329 05	N. 23 10	06	100 32	103 03	105 33	108 04	110 34	113 04
33	*Kaus Aust.*	84 59	S. 34 25	07	115 35	118 05	120 36	123 06	125 36	128 07
34	*Miaplacidus*	221 53	S. 69 27	08	130 37	133 08	135 38	138 09	140 39	143 09
35	*Mirfak*	310 01	N. 49 38	09	145 40	148 10	150 41	153 11	155 41	158 12
36	*Nath*	279 24	N. 28 33	10	160 42	163 13	165 43	168 13	170 44	173 14
37	α Pavonis	54 48	S. 56 56	11	175 45	178 15	180 46	183 16	185 46	188 17
38	*Polaris*	334 20	N. 88 58	12	190 47	193 18	195 48	198 18	200 49	203 19
39	POLLUX	244 37	N. 28 11	13	205 50	208 20	210 50	213 21	215 51	218 22
40	PROCYON	245 59	N. 5 23	14	220 52	223 23	225 53	228 23	230 54	233 24
41	*Ras Alhague*	96 59	N. 12 36	15	235 55	238 25	240 55	243 26	245 56	248 27
42	REGULUS	208 44	N. 12 16	16	250 57	253 27	255 58	258 28	260 59	263 29
43	RIGEL	282 06	S. 8 16	17	265 59	268 30	271 00	273 31	276 01	278 32
44	RIGEL KENT.	141 10	S. 60 35	18	281 02	283 32	286 03	288 33	291 04	293 34
45	*Schedar*	350 45	N. 56 12	19	296 04	298 35	301 05	303 36	306 06	308 36
46	*Shaula*	97 39	S. 37 04	20	311 07	313 37	316 08	318 38	321 09	323 39
47	SIRIUS	259 24	S. 16 38	21	326 09	328 40	331 10	333 41	336 11	338 41
48	SPICA	159 31	S. 10 50	22	341 12	343 42	346 13	348 43	351 13	353 44
49	α Tri. Aust.	109 29	S. 68 55	23	356 14	358 45	1 15	3 46	6 16	8 46
50	VEGA	81 17	N. 38 44	24	11 17	13 47	16 18	18 48	21 18	23 49

SUN, MOON AND PLANETS, 1937 OCTOBER 1 and 2

Friday, October 1

G.M.T. h	SUN G.H.A. ° ′	SUN Dec. ° ′	MOON G.H.A. ° ′	MOON Dec. ° ′
00	182 31	S. 2 56	223 26	N. 8 26
01	197 31	2 57	237 57	8 14
02	212 32	2 58	252 28	8 03
03	227 32	2 59	226 59	7 51
04	242 32	3 00	281 30	7 39
05	257 32	S. 3 01	296 01	N. 7 27
06	272 32	3 02	310 32	7 15
07	287 33	3 03	325 03	7 04
08	302 33	3 04	339 34	6 52
09	317 33	3 05	354 06	6 40
10	332 33	S. 3 06	8 37	N. 6 28
11	347 33	3 07	23 08	6 16
12	2 34	3 08	37 40	6 04
13	17 34	3 09	52 11	5 52
14	32 34	3 10	66 43	5 40
15	47 34	S. 3 11	81 14	N. 5 28
16	62 34	3 12	95 46	5 16
17	77 35	3 13	110 17	5 04
18	92 35	3 14	124 49	4 52
19	107 35	3 15	139 21	4 40
20	122 35	S. 3 16	153 52	N. 4 28
21	137 35	3 16	168 24	4 15
22	152 36	3 17	182 56	4 03
23	167 36	3 18	197 28	3 51

Saturday, October 2

G.M.T. h	SUN G.H.A. ° ′	SUN Dec. ° ′	MOON G.H.A. ° ′	MOON Dec. ° ′
00	182 36	S. 3 19	212 00	N. 3 39
01	197 36	3 20	226 32	3 27
02	212 36	3 21	241 04	3 15
03	227 37	3 22	255 36	3 02
04	242 37	3 23	270 08	2 50
05	257 37	S. 3 24	284 40	N. 2 38
06	272 37	3 25	299 12	2 26
07	287 37	3 26	313 44	2 14
08	302 38	3 27	328 16	2 01
09	317 38	3 28	342 48	1 49
10	332 38	S. 3 29	357 20	N. 1 37
11	347 38	3 30	11 52	1 25
12	2 38	3 31	26 25	1 13
13	17 39	3 32	40 57	1 00
14	32 39	3 33	55 29	0 48
15	47 39	S. 3 34	70 02	N. 0 36
16	62 39	3 35	84 34	0 24
17	77 39	3 36	99 06	N. 0 12
18	92 40	3 37	113 39	— 0 00
19	107 40	3 38	128 11	S. 0 13
20	122 40	S. 3 39	142 43	S. 0 25
21	137 40	3 40	157 16	0 37
22	152 40	3 41	171 48	0 49
23	167 41	3 42	186 21	1 01
24	183 41	3 43	200 53	1 13

MOON P.PS. (same for October 1 and October 2)

G.H.A. m	′
05	1
10	2
15	3
20	4
25	5
30	6
35	7
40	8
45	9
50	10
55	11
60	12

Dec. m	′	′
05	1	1
10	2	2
15	3	3
20	4	4
25	5	5
30	6	6
35	7	7
40	8	8
45	9	9
50	10	10
55	11	11
60	12	12

PLANETS

G.M.T. h	G.H.A. ° ′	Dec. ° ′
Venus, −3·4		
Oct. 1		
00	210 07	N. 9 53
06	300 04	9 47
12	30 02	9 41
18	119 59	9 34
Oct. 2		
00	209 57	N. 9 28
06	299 54	9 22
12	29 51	9 15
18	119 49	9 09
24	209 46	9 03
Mars, 0·3		
Oct. 1		
00	98 51	S. 25 48
06	188 55	25 48
12	278 58	25 47
18	9 02	25 47
Oct. 2		
00	99 06	S. 25 47
06	189 09	25 47
12	279 13	25 46
18	9 16	25 46
24	99 20	25 46
Jupiter, −1·9		
Oct. 1		
00	79 48	S. 22 42
06	170 02	22 42
12	260 16	22 41
18	350 30	22 41
Oct. 2		
00	80 44	S. 22 41
06	170 57	22 41
12	261 11	22 41
18	351 25	22 41
24	81 39	22 41
Saturn, 0·7		
Oct. 1		
00	7 08	S. 1 53
06	97 23	1 53
12	187 39	1 54
18	277 55	1 54
Oct. 2		
00	8 11	S. 1 54
06	98 27	1 55
12	188 43	1 55
18	278 59	1 56
24	9 14	1 56

P.PS.

G.H.A. h m	V. ′	M. ′
0 30	−0	0
1 00	0	1
1 30	1	1
2 00	1	1
2 30	1	2
3 00	1	2
3 30	−1	2
4 00	2	3
4 30	2	3
5 00	2	3
5 30	2	3
6 00	3	4

Dec. h	V. ′	M. ′
1	1	0
2	2	0
3	3	0
4	4	0
5	5	0
6	6	0

G.H.A. h m	J. ′	S. ′
0 30	1	1
1 00	2	3
1 30	4	4
2 00	5	5
2 30	6	7
3 00	7	8
3 30	8	9
4 00	9	11
4 30	11	12
5 00	12	13
5 30	13	14
6 00	14	16

SUN

S.D. 16′·0

MOON

	H.P.
Oct. 1	57′·2
2	56′·7
	S.D.
Oct. 1	15′·6
2	15′·4

Proportional parts are always to be *added* to the G.H.A., except occasionally for Venus.

For the Moon's Dec. use the first column of P.Ps. for the first 12h in each day and the second column for the last 12h.

STAR INDEX

Name	No.	Mag.	S.H.A.	R.A.	Dec.
			° ′	h m	°
β CENTAURI	24	0·9	150 07	14 00	S. 60 05
Benetnasch	18	1·9	153 43	13 45	N. 49 37
SPICA	48	1·2	159 30	13 22	S. 10 51
Alioth	6	1·7	167 09	12 51	N. 56 17
β CRUCIS	25	1·5	168 57	12 44	S. 59 21
Alhena	5	1·9	261 26	6 34	N. 16 27
CANOPUS	21	−0·9	264 21	6 23	S. 52 40
BETELGEUSE	19	*0·8	272 01	5 52	N. 7 24
Alnitak	7	1·9	275 34	5 38	S. 1 58
Anilam	12	1·8	276 43	5 33	S. 1 14
Nath	36	1·8	279 23	5 22	N. 28 33

INTERPOLATION OF G.H.A. ♈

Increment to be added to G.H.A. of First Point of Aries for intervals of G.M.T.

m s		m s		m s	
53		13		32	
	0 14		1 04		1 54
00 57		17		36	
	0 15		1 05		1 55
01 01		21		40	
	0 16		1 06		1 56
05		25		44	
	0 17		1 07		1 57
09		29		48	
	0 18		1 08		1 58
21		41		09 00	
	0 36		1 26		2 16
25		45		04	
	0 37		1 27		2 17
29		49		08	
	0 38		1 28		2 18
33		53		12	
	0 39		1 29		2 19
37		05 57		16	
	0 40		1 30		2 20
41		06 01		20	
	0 41		1 31		2 21

INTERPOLATION OF G.H.A.—SUN AND PLANETS

21^{m}—24^{m}		24^{m}—27^{m}		27^{m}—30^{m}		30^{m}—33^{m}		33^{m}—36^{m}		36^{m}—39^{m}		39^{m}—42^{m}	
m s		m s		m s		m s		m s		m s		m s	
22 02		25 02		28 02		31 02		34 02		37 02		40 02	
	5 31		6 16		7 01		7 46		8 31		9 16		10 01
06		06		06		06		06		06		06	
	5 32		6 17		7 02		7 47		8 32		9 17		10 02
10		10		10		10		10		10		10	
	5 33		6 18		7 03		7 48		8 33		9 18		10 03
14		14		14		14		14		14		14	
	5 34		6 19		7 04		7 49		8 34		9 19		10 04
18		18		18		18		18		18		18	
	5 35		6 20		7 05		7 50		8 35		9 20		10 05
22		22		22		22		22		22		22	
	5 36		6 21		7 06		7 51		8 36		9 21		10 06
26		26		26		26		26		26		26	
	5 37		6 22		7 07		7 52		8 37		9 22		10 07
30		30		30		30		30		30		30	
	5 38		6 23		7 08		7 53		8 38		9 23		10 08
34		34		34		34		34		34		34	
	5 39		6 24		7 09		7 54		8 39		9 24		10 09
38		38		38		38		38		38		38	
	5 40		6 25		7 10		7 55		8 40		9 25		10 10
42		42		42		42		42		42		42	
	5 41		6 26		7 11		7 56		8 41		9 26		10 11
46		46		46		46		46		46		46	
	5 42		6 27		7 12		7 57		8 42		9 27		10 12
50		50		50		50		50		50		50	
	5 43		6 28		7 13		7 58		8 43		9 28		10 13
54		54		54		54		54		54		54	
	5 44		6 29		7 14		7 59		8 44		9 29		10 14
22 58		25 58		28 58		31 58		34 58		37 58		40 58	
	5 45		6 30		7 15		8 00		8 45		9 30		10 15

INTERPOLATION OF G.H.A.—MOON

21^{m}—24^{m}		24^{m}—27^{m}		27^{m}—30^{m}		30^{m}—33^{m}		33^{m}—36^{m}		36^{m}—39^{m}		39^{m}—42^{m}	
m s		m s		m s		m s		m s		m s		m s	
23 03		26 03		29 03		32 03		35 03		38 03		41 03	
	5 31		6 14		6 57		7 40		8 23		9 06		9 49
07		07		07		07		07		07		07	
	5 32		6 15		6 58		7 41		8 24		9 07		9 50
11		11		11		11		11		11		11	
	5 33		6 16		6 59		7 42		8 25		9 08		9 51
16		16		16		16		16		16		16	
	5 34		6 17		7 00		7 43		8 26		9 09		9 52
20		20		20		20		20		20		20	
	5 35		6 18		7 01		7 44		8 27		9 10		9 53
24		24		24		24		24		24		24	
	5 36		6 19		7 02		7 45		8 28		9 11		9 54
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	5 40		6 23		7 06		7 49		8 32		9 15		9 58
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	5 41		6 24		7 07		7 50		8 33		9 16		9 59
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	5 42		6 25		7 08		7 51		8 34		9 17		10 00
53		53		53		53		53		53		53	
	5 43		6 26		7 09		7 52		8 35		9 18		10 01
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